Humanizing Mathematics: An Amalgamation of Constructionist Theory and Situated Cognition in the Mathematics Classroom.

A Dissertation

Submitted to the Faculty

of

Drexel University

By

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In partial fulfillment of the

requirements for the degree

of

Doctor of Education

September 2017

Abstract

Humanizing Mathematics: An Amalgamation of Constructionist Theory and Situated Cognition in the Mathematics Classroom

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Drexel University, September 2017

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A primary contributor to facilitating student learning in mathematics includes the mathematics teacher. The design of instructional delivery, presentation of engaging activities and analysis of student feedback are the key responsibilities that mathematics teachers are tasked with in order to present learning opportunities to the students. The approach to satisfying these responsibilities is contingent on the teachers' values and beliefs regarding the various aspects of mathematical proficiency. In this study, I have investigated teachers' valuation of two of these aspects: procedural fluency and conceptual understanding. The literature on these two aspects of mathematical proficiency has focused almost exclusively on elementary teachers. Research studies have uncovered a proclivity of elementary teachers toward procedural learning despite the abstract nature of mathematical processes. Elementary teachers have been found to possess a certain aversion toward a deeper conceptual understanding of mathematics. With regard to secondary teachers, the research is more focused on instructional strategies that emphasize mathematical concepts, but neglect the relationship that these concepts may have to the robust procedures that allow students to process known values as they attempt to discover solutions to both prescribed and unique problems. This study examined the values that secondary teachers place upon two learning constructs within the framework of mathematical proficiency, and how these values influence their approach to teaching. "How do secondary mathematics teachers value procedural fluency and conceptual understanding in their instructional practices?" is the fundamental question of this study. The study reaches beyond this initial question to also examine the root causes of the establishment of these values by asking "how do teachers develop their value-system of procedural fluency and conceptual understanding in their instructional practices?". This study consisted of two phases. In phase one, teachers completed a 27-item survey instrument in order to gather responses about their beliefs regarding conceptual understanding, procedural fluency and a blend of the two constructs. Phase two consisted of a focus group interview of teachers to delve deeper into the source of the various belief systems that serve as the foundation for mathematics instruction in secondary schools in southeastern Pennsylvania.

This Ed.D. Dissertation Committee from The School of Education at Drexel University certifies that this is the approved version of the following dissertation:

Humanizing Mathematics: An Amalgamation of Constructionist Theory and Situated Cognition in the Mathematics Classroom.

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Dedication

I dedicate this dissertation to my wife, Katie Kennedy-Reilly (Ed.D.), who has been a constant inspiration through her guidance, support, and love as I strive to be the best person and educator I can be. I also dedicate my doctoral work to our four children Maddie, Christian, Sean and Finn, who I hope will follow their own paths in life, and will always work hard to accomplish those pursuits. Finally, I dedicate this dissertation to my parents, G. Patrick and Marge Reilly, who have shown me the honest way to becoming successful - through resilience and perseverance.

Acknowledgements

I offer this special thanks to the many dedicated mathematics educators across southeastern Pennsylvania that contributed to this dissertation by offering their time and perspectives. The data collected from this group of teachers informed the first phase of my study so that I could take the next steps in the process. In particular, I want to thank the six educators who committed an evening of their time to participate in the focus group discussion. The conversational nature of the discussion was enlightening, informative, and a demonstration of the participants' passions for teaching and learning. Their earnest and honest contributions provided various examples and experiences to aid in my pursuit of humanizing mathematics.

Thank you to my committee members Dr. Gould and Dr. McHale-Small for their support, guidance, and encouragement in both the completion of this research study and future studies. I want to especially thank my committee chair, Dr. Reisman for her patience and feedback throughout the process. Dr. Reisman was not only my chair, but also my instructor for several courses on creativity and innovation. Dr. Reisman's work in both education and creativity has been a main source of motivation and encouragement for imparting change in the educational landscape.

Although we took different paths to complete our dissertation work, I want to acknowledge the friendship and love shared with my classmates and program director that defined my experience; thank you Marguerite, Mike, Mike, Dara, Janean, Judy, Kristen, Rashan, Catherine and Dr. Mawritz.

Finally, I would like to thank, with all my heart, my wife and children for their patience, cooperation, encouragement and perspective when trying to find the right balance of family, work, and dissertation. This accomplishment is a tribute to their support.

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Chapter 1: Introduction to the Research

Introduction to the Problem

As with any advancement in society, one must develop the creative and critical skills of adapting procedures for novel problems and unique situations. A student may read a recipe, assemble the ingredients and follow the directions to produce a predictable and palatable entree. This successful endeavor does not bestow the title of "chef" upon the apron of the young master of procedure, although, the achievement is worthy of recognition. Greater acknowledgement of success is earned when the student demonstrates the ability to manipulate the prescribed procedure in order to create a unique meal that extends the foundation of rote techniques and strategies. Whether students are donning their apron in a quest for a culinary creation, or tapping on their calculator in the application of the foundations of Algebra, two learning theories can be recognized as characteristics of the learning process: situated cognition and constructionism (Brown et al, 1989; Lave & Wegner, 1991; Papert & Harel, 1991; Schwartz, 2008).

Situated cognition, or situated learning, is a theory developed by Jean Lave that contends learning is dependent on the activity and social context and culture in which it occurs (Lave & Wegner, 1991). This concept was further developed by John Seely Brown, Alan Collins and Paul Dugid who amended the concept of situated learning with the element of cognitive apprenticeship where experts or teachers model strategies for "using, managing and discovering knowledge" (Brown, Collins & Dugid, 1989). Learning concepts in isolation has described reform in mathematics education since the 1980s (Kilpatrick et al, 2001), but the research conducted by Lave, Brown, Collins and Dugid suggests that learning is social and demands the sharing of knowledge, as well as, collaborative problem solving. The theory of cognitive apprenticeship is meant to "enculturate students into authentic practices through activity and social interaction in a way similar to that evident – and evidently successful – in craft apprenticeship" (Brown et al., 1989). Teachers and learners are engaged in a symbiotic relationship of scaffolded and contextual education.

The second theory of learning, presented by Seymour Papert, is constructionism. Constructionism is also a social learning theory, and is rooted in Jean Piaget's philosophy of education known as constructivism. Both Piaget and Papert believe that knowledge is actively constructed by the student, but the manner and condition in which that construction occurs varies between the philosophy and the learning theory. In order to maintain parallelism between the title educational theories, Papert's constructionism as opposed to Piaget's philosophy of constructivism will be utilized as a foundation for the investigation being proposed. Piaget's constructivism describes learning as a process of interpretation within a framework of existing knowledge and experience among students. As students build their knowledge, they progress along predetermined developmental stages. An important condition in the theory of constructivism is the internalization of learning that occurs as students organize and reorganize their knowledge with each new experience they interact with in the developmental process.

This notion of internalization is critical in the distinction between constructivism and constructionism. Papert and Harel (1991) state the following:

> Constructionism – the N word as opposed to the V word – shares constructivism's view of learning as 'building knowledge structures' through progressive internalization of actions … It then adds the idea that this happens especially felicitously in a context where the learner is consciously engaged in constructing a public entity, whether it's a sand castle on the beach or a theory of the universe (p. 1).

Papert emphasizes the need for students to express their ideas as “tangible and shareable which, in turn, informs, shapes and sharpens these ideas” so that they can be communicated with others (Ackerman, 2001, p. 4). Constructionism considers learning to be an external exercise that is both contextual and sensible. These conditions of learning are similar to the concept of situated learning, and therefore, the inclusion of constructionism is more precise than that of constructivism in this study. While Piaget examined learning as a progressive detachment from concrete expressions of learning to symbolic representations, Papert’s idea of learning relies on the connectedness of students to the situation or context of the learning environment (Ackerman, 2001).

Papert’s concept of learning demands students to associate mental models acquired through discovery learning with authentic experiences in building of tangible and meaningful physical models. Like the student learning to cook, instructional practices that merely share knowledge in a mathematics classroom will result in isolated recipes of knowing disconnected from students’ ability to perform mathematics in unique situations. Papert relates this message to science, although it should be considered ubiquitous throughout schools: “telling children how scientists do science does not necessarily lead to far-reaching change in how children do science; indeed, it cannot, as long as the school curriculum is based on verbally-expressed formal knowledge” (Papert & Harel, 1991). The need for such development in the mathematics classroom has been discussed for nearly two decades now, yet math education is still fixated on procedural learning rather than conceptual understanding and procedural fluency (Anderson et al, 2015; Kilpatrick et al., 2001; NAEP, 2015; NRC, 2005; OECD, 2013; Schoenfeld, 2013; TIMMS, 2015).

Mathematics proficiency is characterized by conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick et al., 2001). The first two components are the focus of this study, and are defined below for the purpose of clarity. Kilpatrick et al. present conceptual understanding as the "comprehension of mathematical concepts, operations, and relations" (2001, p. 5). Conceptual understanding, also characterized as heuristic learning, is identified by high level thinking intended to form abstract representations of the structures that guide mathematical discovery, while also establishing relationships among those structures. Procedural fluency is the skill of "carrying out procedures flexibly, accurately, efficiently, and appropriately" (Kilpatrick et al., 2001). Procedural fluency, also considered algorithmic learning, has been traditionally regarded as the principal approach to K-12 mathematics education. The focus of learning is grounded in skill development with a secondary concern for increasing efficiency.

A perpetual focus among public education institutions is on the narrowing of the achievement gap through focused attention on differentiating instructional delivery of reading and mathematics knowledge and skills. From the origins of John Dewey's educational theories and philosophies, current psychologists have reinvigorated the notions of student-centered experiential learning. No-Child-Left-Behind and the more recent Common Core promote standardization of education, which has been highly regarded by some and met with contempt by others. Standardization remains synonymous with traditional teaching and static classrooms. Dewey (1933), Jean Piaget (1937) and Lev Vygotsky (1978) each contributed to the concepts of student-centered, flexible and authentic learning opportunities as the foundation for providing a truly participatory education for students. Despite the perpetual lessons offered to pre-service teachers on the validity of constructivism, social learning and situated cognition, many elementary and secondary math classrooms rely on rote memorization and predictable application of numeracy, operations and patterns.

Aside from the classical educational psychologists, Henri Poincaré and George Polya offer specific contributions to the mathematics arena of the education industry. These two mathematicians offered learning theories that related logic to the creative "aesthetic" of mathematics and structure to problem-solving techniques, respectively. Seymour Papert's work in constructionism is greatly influenced by both Poincaré and Polya (Papert, 1980). Additionally, contemporary publications by Gray et al. (1999), Reisman and Torrance (2000 and 2002), Schoenfeld (2013), and Schwartz (2008) borrow the conceptual foundations of these historical figures in mathematics education to promote new instructional strategies that emphasize the symbiosis of creativity and student engagement through problem solving.

Traditional mathematics education has an overwhelming dependence on convergent thinking (Kilpatrick, 2001; NMAP, 2008). This pathway to learning is analogous to procedural learning or an overemphasis of procedural knowledge which is not simply knowing how to perform a variety of procedures, but also knowing which procedures or strategies are appropriate for a particular situation (NRC, 2005; Star, 2005). Instructional strategies and student exercises that support procedural learning are valuable, but present barriers to a more creative approach to learning mathematics. These barriers lead children to become risk averse and focused on good grades rather than learning (Runco, 2014; Sternberg & Lubart, 1995). Although there are several avenues for infusing creativity into mathematics lessons, problem solving experiences require a broad depth of knowledge, an artful skillset in inductive reasoning, and an intrinsic motivation to exercise one's creative potential. When teachers allow for opportunities of creative thinking in their lessons, they aid students' avoidance of rote learning and evade the trap of being blinded to learning possibilities by one's still developing mathematical schemata (Frensch & Sternberg, 1989; Runco, 2014; Simonton, 1984; Torrance & Reisman, 2000). Both the former and latter results of student practice of creativity in mathematics yields this second component of the symbiosis: the notion of student engagement.

In research conducted by Anderson, Valero and Meaney, "bored was a word used most often by 16-year-old students to describe their attitude in a questionnaire on their affective relationship to mathematics" (2015). Instructional strategies involving creativity and problem solving requires students to absorb procedural knowledge, process that new knowledge, and then utilize that knowledge as a means to progress. The active involvement in constructing one's own education provides "insight into reasons for engaging in a particular area of study, encourages activity as opposed to passivity, and provides opportunities to be the doer" (Reisman & Torrance, 2002, p. 30). The overarching themes of constructionism and situated cognition are both representative of the nature of student engagement through problem solving experiences that celebrate – rather than dilute – students' creativity. According to the National Mathematics Advisory Panel (NMAP), an emphasis on effort over ability is "related to greater engagement in mathematics learning" (2008). By situating students in a learning environment where they are faced with unique challenges that demand access to prior knowledge, a contextual association between the present situation and their own knowledge base, and the virtues of collaboration, students become engaged with the learning process first, and subsequently demonstrate their learning through the active construction of mathematical models representing their newly created understanding of the learned concept (Lave & Wenger, 1991; Papert, 1980; Papert & Harel, 1991).

These concepts are abstractions though. There are various avenues that may lead to this level of understanding, and creative problem solving is just one method. Additionally, the strength of foundational knowledge must first be established. The challenge in learning mathematics resides among a variety of considerations that are unique to each student. When approaching their instruction in the vain of constructionism and situated cognition, teachers need to be mindful of the learner's background, their attitudes toward the content area, their emotional stability when experiencing errors, their intrinsic and extrinsic motivators and their previous experience with particular procedures and concepts (Reisman, 1982). Students should be encouraged and allowed to put their newly learned mathematics to use (Brownell and Hendrickson, 1950) in order to retain the concept, which can then be utilized as a foundation for a new progression in their learning. If, on the contrary, students suffer from inaction and gather a mounting summit of unused knowledge, the foundation cannot remain stable and supportive of progress, and retention of the concept is unlikely (Reisman, 1982).

This research study was aimed at exploring the phenomenon of the amalgamation of two learning constructs in the Pennsylvania public school system of secondary mathematics: procedural fluency and conceptual understanding. This exploration was guided by Poincaré's theory of mathematical aesthetics. As presented by Papert, the first stage of Poincaré's theory requires a "deliberate, conscious analysis" (Papert, 1980). Regardless of a students' procedural fluency or conceptual understanding, the problem may be decidedly too difficult to yield a solution. In the constructionist approach the teacher's role, then, is to develop the useable knowledge necessary to construct the solution.

The teacher's approach to instruction is the centerpiece of this study. Traditionally mathematics has been regarded as depersonalized, although the more recent mathematics lexicon includes “understanding” and “discovery” (Anderson et al, 2015; Ozgun-Koca & Sen, 2011; Papert, 1980). The syntax of the subject does not necessarily render pedagogical enhancement or enrichment. The value of retention of student learning is tested continuously, and an understanding of how teachers value the construction of students’ reality as it relates to mathematics wa s one objective in this exploration of the learning phenomenon. From the philosophies of John Dewey, Jean Piaget, and Lev Vygotsky, pre-service teachers’ introduction to the profession of teaching is grounded in both philosophy and psychology. In particular, teachers of mathematics should also isolate the contextual beliefs of Poincaré and Polya as they pay explicit attention to not only the content they teach, but also the algorithmic and heuristic balance with which they instruct.

Statement of the Problem to Be Researched

Students engaged in secondary mathematics classrooms are being subjected to a barrage of information and computational strategies that are being regarded as confusing, unnecessary or disconnected among the students (Alon, 2012; Anderson, 2015; Gray et al, 1999; NMAP, 2008; Papert, 1980). Teachers are able to deliver the algorithmic content, but without the heuristic muscle necessary to make the ideas stick. Misconceptions among the different components that define mathematics proficiency appear to be pervasive. As previously noted, mathematics proficiency is characterized by conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick et al., 2001). This study focused on the valuation that teacher’s place on the first two components: conceptual understanding and procedural fluency.

Purpose and Significance of the Problem

Purpose Statement

The aim of this study was to uncover the root of the discontinuity that exists between the current mandate for a balanced approach to mathematics education and the actual instructional practices of secondary mathematics teachers. The National Council of Teacher of Mathematics (NCTM) and the National Research Center (NRC) each endorse the teaching of skills for solving algorithms while also identifying and promoting the conceptual understanding that adds credibility and validity to the math topics being taught. In 1999, Hiebert reported students' motivation to understand mathematical procedures is diminished once they have memorized and practiced the procedures regardless of their level of conceptual understanding. NCTM concludes that students' progress in conceptual understanding should not consist of isolated experiences from procedural exercises, but rather in conjunction with instruction on procedures (2014). Ultimately, the robust development of both procedural skills and conceptual understanding will lead to proficiency in procedural fluency where students have the ability to adapt previous learning to unique and authentic situations.

Significance of the Problem

Math instruction is marred by the isolation of drill and practice techniques that obscure the servitude of math concepts, such as positive and negative integers, numeracy, and arithmetic in more byzantine real-world problems. For example, Alon (2012) studied the teaching and learning of fractions in an elementary classroom. Alon found that the minimalistic approach of direct instruction followed by derivative manual practice did not afford the students significant learning for positive retention of concepts related to fractions. Isaacs and Carroll (1999) affirm that too much practice too soon can be ineffective or lead to math anxiety. Teachers' insecurity toward mathematics instruction in elementary schools is hampering the development of a necessary strength of foundational ability and resilient problem-solving attitude toward learning among young math students. The National Mathematics Advisory Panel (NMAP) has identified the following characteristics of how children learn as a guide for teachers to recognize: a) the advantages for children in having a strong start; b) the mutually reinforcing benefits of conceptual understanding, procedural fluency, and automatic (i.e. quick and effortless) recall of facts; and c) that effort, not just inherent talent, counts in mathematical achievement (2008). Teachers' efforts to influence positive student engagement in mathematics can be considered a blend of these three characteristics, a commitment to resolute problem solving, an emphasis on critical and creative thinking, and opportunities to construct learning in authentic situations.

21st Century learners are situated in instructional organizations focused on a standardized education. These students are being taught crucial skills for their future success in a global marketplace, but the transferability of these skills to unique and authentic situations may be lost among their own dispirited attitude toward learning. Often times the primary emphasis of math instruction and student activity involves explicit procedures that lead to a parade of calculations intended to produce correct answers to mathematical statements. The learning is not necessarily rooted in authenticity or meaning on the contextual level of the students. The instruction is not connected to schemata that students have experienced, and therefore, they may feel more perplexed as they exit class compared to when they entered. Mathematics teachers are trained to present students with opportunities to engage with the content through demonstration and practice of the applicable algorithms. This approach consumes a majority of the class time, leaving little to no room for helping students to make connections between the procedural skills and their conceptual meaning. The enduring task of a math teacher is to package the foundational content in a cloak of authenticity and applicability with the security of confidence in their own ability both to "do" math and "teach" math.

Although many of the challenges relative to positive student attitudes toward mathematics have their origins in elementary schools, the focus of much of the research surrounds Algebra. Throughout this study the term algebra is used to include secondary school algebraic material independent of specific course work entitled Algebra (NMAP, 2008). NMAP has identified Algebra as a "central concern" because of the empirical data that indicates a "sharp falloff in mathematics achievement" coinciding with the start of algebra course work in middle school (2008). As students progress in their mathematics experience, algebra content serves as the foundation for both higher levels of math, as well as, success in college and career earnings (NMAP, 2008). The National Mathematics Advisory Panel warns that "without substantial and sustained changes to its educational system, the United States will relinquish its leadership in the 21st century" (2008). This prediction has become a reality when considering the 2012 Programme for International Student Assessment (PISA) data for mathematics and the 2011 8th grade Trends in Mathematics and Science Study (TIMMS) data. The United States was ranked 26th out of 34 countries on the 2012 PISA and between 7th and 10th out of 34 on countries on the 2011 TIMMS; the reason for a range of rankings in the TIMMS is that the TIMMS data reports a percentage of students scoring at four different levels. The United States' position in the national and international rankings for mathematics achievement has ratcheted down or has remained relatively stagnant at best, and reformation among math teachers is one approach being suggested by the NMAP to reverse the trajectory of the performance of our students.

The research surrounding the necessity of order to conceptual understanding and procedural fluency has developed from a growing trend in mathematics education requiring the re-teaching of material year after year (Anderson et al, 2015; Arslan at al, 2012; Bahr & Bossé, 2008; Kilpatrick, 2001; OECD, 2015; Schwartz, 2008). The National Mathematics Advisory Panel calls for a “focused coherent progression” so that students can continually develop their math skills through constructionist exercises rather than rote algorithmic memorization (Moldavan, 2008; NMAP, 2008). As evidenced through the researcher’s experience, math students are engaged in learning that is mechanical, and sometimes robotic, as a result of programmatic pedagogy that neglects authentic learning (Beilock et al, Liu & Thompson, Peker & Ertekin and Tseng et al). The researcher considers the National Mathematics Advisory Panel’s call for progress to include delivery of instruction with the objective of learning synergy between conceptual understanding and procedural fluency. The purpose of this study was to examine the perspectives of secondary mathematics teachers regarding the acceptance and adoption of this notion of learning synergy.

Research Questions

The purpose of this research study was to examine the consistency of an equivalent attribution of the two learning constructs prescribed by procedural fluency and conceptual understanding through the delivery of mathematics instruction in secondary public schools. This mixed methods investigation was conducted with a phenomenological lens, and directed by both a quantitative exploration and qualitative investigation of secondary mathematics teachers’ perspectives practicing in grades seven through twelve across several suburban school districts in southeastern Pennsylvania (PDE, nd).

This study examined teachers’ perspectives regarding the necessity of both procedural fluency and conceptual learning in courses emphasizing Algebraic skills. The research questions that were used in the study provided a quantitative depiction of teachers’ valuation of the two learning constructs and a qualitative description of how and why teachers interpret the nature of the mandated learning constructs as they pertain to instructional delivery of Algebraic concepts. Through the use of a survey instrument, participants provided feedback on each construct individually, as well as, collectively and with reference to the necessity of learning synergy in secondary mathematics. In a second phase, the researcher conducted a focus group interview consisting of a sample of the surveyed participants. The focus group was intended to further explore the beliefs and rationale for those beliefs that teachers possess relative to the topic of procedural fluency and conceptual understanding in secondary mathematics.

The researcher employed a sequential explanatory mixed methods approach to the design of this study. Phase one of the study consisted of a survey with questions that are categorized by each of the two learning constructs: a) procedural fluency and b) conceptual understanding, as well as, a blend of the two. The survey is not an original design of the researcher, but the categories determined by the designer had been confirmed to associate with the constructs presented for the purposes of this study. The researcher had discussed the availability and applicability of the survey with the creator of the instrument through both electronic communication and telecommunication. The survey is a 27-statement instrument that has two distinct, if not dichotomous, categories consistent with the learning constructs of procedural fluency and conceptual understanding. The survey was designed to provide results that would answer the central research question below. Consistent with a sequential explanatory mixed-methods design, the researcher analyzed the phase one data prior to progressing to phase two (Cresswell, 2009). Phase two consisted of a focus group that included participants randomly selected from the data collected in phase one. The focus group protocol included five questions designed to address the elements regarded in the two sub-questions below. The proceedings for this study and subsequent data analysis were directed by the following central question and two sub-questions:

Central Question: how do secondary mathematics teachers value procedural fluency and conceptual understanding in their instructional practices?

Sub-question 1: how do secondary mathematics teachers' valuation of procedural fluency and conceptual understanding influence their pedagogical decisions?

Sub-question 2: how do teachers develop their value-system of procedural fluency and conceptual understanding in their instructional practices?

Conceptual Framework

Researcher's Stances

The approach to this study of teachers' perspectives on mathematical practices was a blend of situated cognition and constructionism. The study of teachers' perspectives was embedded in the social, cultural and historical construction of their individual reality. The roles and responsibilities of a teacher, both in and outside the classroom, provided insight into the debate between advocates of procedural learning and conceptual understanding. As a father, I have witnessed a paradigm shift in math education as demonstrated in the elementary classrooms that my daughter participated in during the early 2000s compared to those that my youngest son is engaged in today. In the earliest part of the new millennium, students were learning math through direct instruction focused on the process of computation. Students were able to solve scripted problems using fundamental skills that would later serve as the building blocks to more complex and more complicated math situations. The test of learning in this model is not realized until students reach higher level math courses, typically not occurring until high school. The gap in time presents the potential for a significant loss in learning opportunity.

As a high school math teacher, I observed students who can perform complex calculations with superior accuracy, but little understanding of the result. Furthermore, although these students can perform the intended computation, they lack the ability to reason why the computation is both appropriate and accurate. Secondary mathematics programs are focused on specific algorithmic context for a particular topic, but lack opportunities of a more heuristic nature. A modern spiral curriculum, such as Everyday Math, is an example of a program intended to balance student learning in both constructs of conceptual understanding and procedural fluency.

The value of a synergistic approach to learning mathematics was witnessed during my experience as a Research and Development Engineer. My responsibilities included problem finding, problem solving, product invention, and manufacturing innovation. These different tasks are aligned with the intent of the current Standards of Mathematical Practice and the Common Core Standards for Mathematics. In the authentic example of an engineer, a balanced education in mathematics should encompass all aspects of a situation, including a) procedures, b) processes, c) applications, d) operations, and e) deep understanding of why the situation has occurred and how a resolution will impact an individual, an organization, or society holistically.

Conceptual Framework

The research study has been organized around the following three themes: 1) learning culture, 2) conceptually-based learning theories: the investigation of conceptually-based intervention strategies to help promote a balance between algorithmic and heuristic learning, and 3) the nature of mathematics learning: an examination of the nature of mathematics learning in both the procedural and conceptual arena.

Research has been found to recognize that math students' engagement in their learning is dependent on the attitudes and activities of the classroom teacher. In studies by Peker and

Ertekin (2011) and Beilock et al. (2010) it was found that teachers' math anxiety and math teaching anxiety have an adverse effect on student learning, as well as, students' attitudes toward mathematics. This research is generally limited to elementary teachers, and does not provide information regarding student attitudes as they progress from the elementary grades. Additionally, Tseng et al. (2011) and Liu and Thompson (2004) discovered that a strict emphasis on algorithmic teaching and learning neglected aspects of social constructionism that included empathy, collaboration and incrementation in the learning process. The results of this single-dimensional approach to instruction minimized student engagement and diminished excitement toward learning the content.

Research on a variety of intervention strategies has revealed some positive significance of technologically-driven activities in student achievement in math education. This research has generally been limited to secondary and post-secondary students. Additionally, the activities are typically electronic in nature. Furthermore, Moldavan (2008) and Byrnes and Wasik (1991) reported teachers have been found to rely on algorithmic strategies because of their own aversion to conceptual understanding in mathematics. Initially, the need for such intervention strategies developed from the recognition of such an aversion. This research study examined the influence of teachers' valuation of conceptual understanding and procedural fluency on decisions regarding instructional strategies, activity design and assessment administration. In addition to investigating teachers' beliefs, the rationale for these beliefs was queried in order to examine the relationship between how the teachers learned versus how the teachers teach.

Research has addressed the issues of procedural fluency and conceptual understanding, both in isolation and in conjunction. Previous studies by Bednar and Sweeder (2010), Canobi (2008) and Byrnes and Wasik (1991) report on a wide range of mathematical skills and grade levels relative to the incorporation of algorithmic and heuristic learning. The literature raises several philosophical issues surrounding the interrelationship of the two learning constructs. Despite the vastness of this research, the literature, at times, is limited in the connections formed with previous educational psychologies. Additionally, the research is inconclusive as several authors have posited disparate reports when regarding the literature holistically.

In order to address the general concern of the lack of conceptual understanding in mathematics, the researcher has identified three themes to be explored. The first theme relates the learning culture within mathematics that is formed by the perspectives of teachers regarding the utilization of algorithmic and heuristic strategies. The second theme involves the teachers' experience and subsequent emphasis on conceptual activities and assessments that results in non-pedagogical influences on students' learning in the math environment. The third, and final, theme explores the relationship between procedural fluency and conceptual understanding, and if that relationship is reciprocal or even necessary in the learning of mathematics.

Additionally, there are other issues that impact student learning and the design process for this research study. Common Core Standards still have to be met. More specifically, the NCTM Math Practices guide the classroom teacher in lesson design and delivery. The specific content that must be afforded the students resides in these documents. So far, the research studies found typically do not address the standards that are being covered during the intervention activities. This research study addressed the role of standards in experimental design related to the proposed topic by examining the National Standards for Mathematical Practice.

Finally, several of the studies are introduced with a historical setting related to the various psychologists' theories, the advent of adopted regulations, or attempted and successful educational reforms (Edens & Potter, 2012; Francisco, 2013; Tall, 2008; Tseng et al., 2011). The researcher has found the depth of the research to reside in the frequent references to different educational theories stemming from Jean Piaget, Lev Vygotsky and John Dewey. Additionally, the concentration on mathematics has uncovered the works of Henri Poincaré's emotional connection to mathematics revealed by the aesthetic or intuition of the content, and George Polya's emphasis and extension of mathematical problem-solving that is a prerequisite to the construction of student learning.

The purpose of the study was to identify the valuation teachers' have toward conceptual understanding and procedural fluency, and how their beliefs impact pedagogical decisions so that students' questions of "why am I learning this" may begin to be answered. Based on the author's experience, the theories of the aforementioned mathematics and psychology experts have been lost in the minutia of curriculum and standards. A return to their proposals of "theory of knowledge" may provide the path that the researcher seeks through the literature. In Figure 1-1, the relationship among the various concepts (or theories of knowledge) influencing mathematics education are depicted.

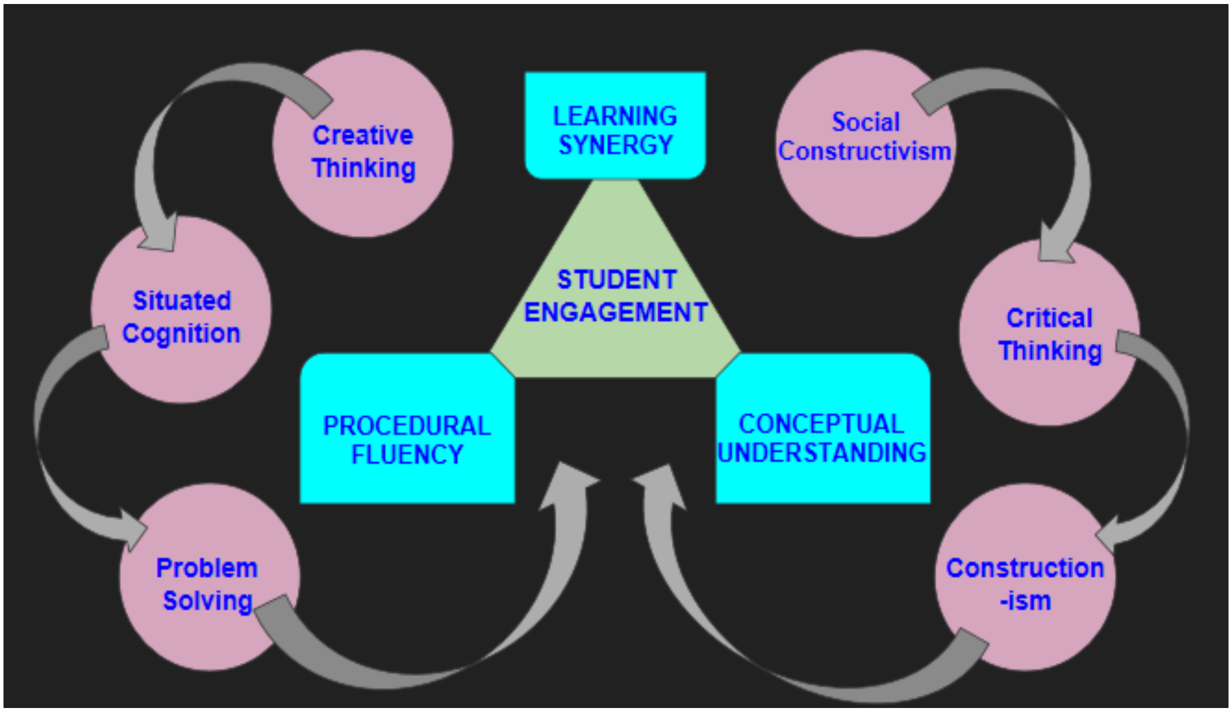

Figure 1-1: Learning Theories for Student Engagement

Definition of Terms

Contextual terminology will be used throughout this study to define the problem and purpose, relate the methodology to the intent of the study, and synthesize the data to be utilized in the discussion of the results and findings. The following list provides specific terminology and the associated meanings for the purposes of consistency and clarity.

Algorithm – a precisely-defined sequence of rules telling how to produce specified output information from given input information in a finite number of steps (Kilpatrick et al., 2001).

Conceptual Understanding – comprehension of mathematical concepts, operations, and relations (Kilpatrick et al., 2001).

Constructionism – learning theory where students learn most effectively through the "making of" tangible objects in the real world (Papert & Harel, 1991).

Procedural Fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately (Kilpatrick et al., 2001).

Procedural Learning – students learn step-by-step processes to accomplish a mathematical task. Procedural learning is also related to rote learning of algorithms (NCTM, 2000).

Proceptual Learning – a mathematical construct in mathematics of an amalgam of the process that produces an object and a mathematical symbol that represents the process or the object (Tall, 2008).

Situated Cognition – learning theory that emphasizes the student's relationship with "knowing" and "doing" meaning that the acquisition of knowledge is situated in a socially, culturally and physically contextual activity (Brown et al, 1989; Lave & Wegner, 1991).

STEM – Science, Technology, Engineering and Mathematics education program. STEM serves as a proxy for the "innovation economy" – the nexus of scientific discovery, innovations, and the commercialization of these innovations into products and business models that help in the economic success in the global marketplace (Meeder, 2013).

Assumptions and Limitations

Assumptions

This research study was designed with assumptions regarding 1) teachers' proclivity toward math, 2) the viewpoint of secondary math teachers regarding instructional practices particular to algebraic math content, and 3) the expected results of the chosen research design. Researchers (Peker and Ertekin, 2011 and Beilock et al., 2010) have reported that elementary teachers experience a higher degree of anxiety toward mathematics than any other content area during their pre-service training. No such information regarding secondary mathematics teachers had been discovered by the researcher. The perspectives of secondary teachers regarding their valuation of procedural fluency and conceptual understanding were examined in this study, and the researcher assumed that the anxiety levels of secondary teachers is not consistent with those in elementary schools. This assumption suggests that the participating teachers were both confident and capable in evaluating their perspectives of the subject learning constructs.

Secondly, secondary mathematics teachers are specialized to teach the entire breadth of Algebra related skills. It was assumed that each participant is experienced and certified in teaching secondary mathematics, and is therefore familiar with the content standards. As an extension of this consideration, the researcher viewed each participant as an expert in their domain who could accurately recognize student engagement through participation in instruction, learning activities and assessments.

Finally, the researcher assumed that the mixed methods design will yield sufficient and significant data necessary for addressing the topic of the study.

Limitations

It is expected that the curriculum being delivered in the participating secondary schools is designed to satisfy the Standards for Mathematical Practice. Although many schools function with standards-based classrooms, it is unknown to the researcher if all schools have adopted this philosophy. The research being proposed will examine the efficacy of teacher reported instructional practices as they relate to the national standards, and is contingent on honest and accurate self-reporting through the use of a survey instrument.

The sample of participants were secondary mathematics teachers who had experience teaching Algebra related skills. The study was meant to examine all eligible secondary mathematics teachers' viewpoints, and therefore, the level of experience with teaching

Algebraic skills was expected to be variable, and the impact of this variability on the findings of the study was acceptable to the researcher.

The nature of purposive sampling, specifically snowball sampling, yielded a small sample size. The researcher's position as a high school mathematics teacher allowed access to participants that satisfy the prescribed criteria. Utilization of these "participants to identify additional cases who may be included in the study" did not guarantee a known sample size (Teddlie & Tashakkori, 2009).

The testing instrument for this mixed methods study was a web-based survey. The exploration of teacher perspectives of procedural fluency and conceptual understanding would be ideally studied through observation. The timing of the study relative to access to the teacher participants prohibited the researcher from engaging in a methodology that included observation.

Delimitations

The entirety of this study took place across several school districts in southeastern Pennsylvania. This constraint on the geographic placement of the study suggested the findings may be generalizable only to other school districts in Pennsylvania.

The researcher chose to conduct the study with an emphasis on secondary mathematics, particularly Algebra related content. The research acknowledged that the methodology could have been altered to include other mathematical topics, and different stages of education.

Summary

This chapter presents an overview of the purpose and rationale for examining the balance of procedural fluency and conceptual understanding in the mathematics classroom. The researcher investigated the values of teachers of secondary mathematics with a focus on Algebraic skills to attempt to discover a relationship between current instructional practices and their emphasis on either procedural fluency, conceptual understanding or an amalgamation of the two. The collection and subsequent analysis of the data produced an understanding of how teachers utilize the learning constructs of procedural fluency and conceptual understanding in their daily interaction with students engaged in Algebra content. The second phase of the study was aimed at a rich exploration of the teachers' rationale for either a particular emphasis on one construct over the other, or a more blended approach.

The following chapter provides additional background information collected from a variety of sources. The research of existing literature has guided this study in both its content and it's stylistic approaches to the various themes that have been collated; these themes involve : 1) the relationship between teachers' content and pedagogical knowledge and the learning culture, 2) the investigation of conceptually-based intervention strategies to help promote a balance between algorithmic and heuristic learning, and 3) an examination of the nature of mathematics learning in both the procedural and conceptual context. Subsequent to a review of the literature, Chapter 3 provides the details of the completed study regarding the participants, site selection, methodology, and ethical considerations. The action plan entails specific descriptions of the data collected, and the methods of analysis employed.

Chapter 2: The Literature Review

Introduction

The purpose of the literature review was to assemble the relative studies that have already been conducted on topics related to teachers' valuation of procedural fluency and conceptual understanding in the mathematics classroom. The purpose of this chapter was intended to collect a plethora of background information to help shape the research study so that the gaps in the literature can begin to be closed, and the author's contribution to the body of knowledge can aid in future research and continual propagation toward the "ideal" education plan for authentic learning.

The research was focused on identifying the necessary components to a thorough mathematics education that both satisfies the progressive steps of knowledge for continuous development of the student, as well as, provides authenticity to the learning episodes fulfilling the students' inquiry into the purpose behind their education. The researcher identified the areas of learning culture, learning theories and the nature of mathematics education as major facets of exploration for the study.

The research surrounding the necessity to focus upon conceptual understanding and procedural fluency has developed from a growing trend in mathematics education requiring the re-teaching of material year after year. The National Mathematics Advisory Panel calls for a "focused coherent progression" so that students can continually develop their math skills through constructivist exercises rather than rote algorithmic memorization (Moldavan, 2008; NMAP, 2008). Through an investigation of the literature, the researcher uncovered the impact that mechanical, and sometimes robotic, pedagogy in the classroom has on authentic learning (Beilock et al., 2010; Liu & Thompson, 2004; Peker & Ertekin, 2011; Tseng et al., 2011).

Generally, the research has been more explicit in the description of the instructional and assessment strategies used (Allen, nd; Alon, 2012; Bednar & Sweeder, 2010; Byrnes & Wasik, 1991; Canobi, 2008; Daher & Jaber, 2010; Edens & Potter, 2012; Gogus, nd; Liu & Thompson, 2004). The issue of instruction is separated into two categories: traditional and experimental. The research studies that have been identified as experimental in nature have presented the different strategies used (Cai et al, 2013; Francisco, 2013; Gray et al., 1999; Kang, 2007; Mandrin & Preckel, nd; Schoenfeld, 2013). Some studies are experimenting with a new strategy to impact student learning, while others use traditional direct instruction presentation strategies to deliver material (Carr, 2012; Ernst & Clark, 2012; Kabapinar, 2005; Moldavan, 2008; Nicoll-Senft, 2009; Rogers & Portsmore, 2004; Thornburg, 2013). It is important to note the different strategies used when considering either replicating a study, or attempting to refute the results of a particular experiment.

Despite efforts to standardize education reflected in the mandates of the No Child Left Behind (NCLB) Act of 2001, modern day psychologists have emphasized the need for student-centered experiential and traditional constructivist learning. Standardization alludes to stagnant and life-less learning among today's mathematics students. Educational pioneers such as John Dewey, Jean Piaget and Lev Vygotsky established their own version of a student-centered, flexible and authentic learning environment as the foundation for providing a truly participatory education for students. Recent research borrows the conceptual thinking of these historical figures in education to aid in the development of more innovative instructional approaches. Each piece of the research forms a bond between or among two or more of these theorists. It is the holistic perspective that may be necessary to propagate the work that is being proposed in this study.

Although the literature is rich with data, theories and intervention strategies, there are areas that have not yet been reported on. Most glaringly is an examination of the constructs of conceptual understanding and procedural fluency in conjunction with various learning theories while regarding teachers' beliefs. More specifically, each theme explored has research isolated in its use of either grade level, mathematical content or intervention strategies. Previous research that is focused on the impact that new intervention strategies has on mathematics education has not examined the theories of Seymour Papert's constructionism or Jean Lave's situated learning. The goal of the author was to examine the relationship between teachers' valuation of conceptual understanding and procedural fluency by inquiring about the learning opportunities provided in the context of these two theories.

Literature Review

The concepts and conclusions of previous research are presented on the following pages. The literature reviewed has been organized into three themes: a) the influence that teachers' beliefs has on the learning culture, b) the integration of various learning theories and instructional strategies in mathematics education with specific regard to concept development, and c) the importance of both procedural fluency compared and contrasted with the importance of conceptual learning as natural component of mathematics education.

Theme 1: The Learning Culture

There are two major considerations regarding how teachers' beliefs influence learning. The first is the relationship between teachers' attitudes and perceptions of their own ability to convey mathematics content in classroom settings and the resulting student attitudes toward mathematical concepts. The second consideration relates instructional strategies, specific content tasks and informal classroom activities to students' attitudes and achievement in learning mathematics.

Teachers' pedagogical skills and content knowledge are often regarded when determining their effectiveness in a classroom setting. Recent studies have found that a teacher's attitude and comfort level with teaching a particular content also impacts students learning. In particular it has been reported that female elementary teachers have the highest level of math anxiety of any college major (Beilock et al., 2010; Peker & Ertekin, 2011). Anxiety towards mathematics performance correlates directly with low mathematics grades, failure to enroll in high level math courses, poor performance on standardized tests, and possibly failure to graduate from high school (NMAP). This revelation suggests that a change in teacher preparation programs may be necessary to improve teachers' self-efficacy prior to their initiation into the classroom. The empirical data provided in a study by Peker and Ertekin (2011) identifies four factors of anxiety, and signifies a positive correlation between math teaching anxiety and math anxiety. The four factors of anxiety are: anxiety caused by content knowledge, anxiety caused by self-confidence, anxiety caused by attitude towards teaching mathematics and anxiety caused by methodological knowledge. Although these specific factors were not specified when examining student anxiety, Beilock et al. (2010) discovered that female students in agreement with a traditional stereotype of poor mathematic ability among females had lower math achievement than female students who refuted the stereotype.

The National Research Council (NRC) confounds the issue of teachers' non-pedagogical influence on student learning by also identifying curriculum content, learning processes and teachers' education as contributors to students' challenges to learning (Moldavan, 2008; Byrnes

& Wasik, 1991). Specifically, a teachers' pre-service record can suggest a linkage to their inability to convey information on a conceptual basis. Ma (1999) discovered that many US teachers were able to perform mathematical tasks on an elementary level, but could not explain the conceptual understanding necessary to validate the procedural methods employed. A teacher's lack of self-efficacy and lack of heuristic understanding in the area of mathematics based on their teacher preparation program are major factors in the NRC's investigation into high quality instructional practices (Moldavan, 2008; Kajander, 2010). Although elementary teachers have been found to be at the forefront of these discoveries, the delivery of content has not necessarily impacted students' procedural learning. The difference between the two learning domains resides in the concept of retention and application of the material learned. Procedural learning concerns the process of problem solving, which consists of identifying a known input applying a mathematical operation to that input to achieve a predictable output. Conceptual understanding extends the learning to include the application and synthesis of many processes to create new unpredictable solutions to unique complex authentic problems (Gray et al., 1999).

In studies conducted by Kang (2007) and Francisco (2013), teachers participated in experiments investigating the influence of group work and change pedagogy. In both instances, the role of the teacher was altered in order to measure the effects of removing the instructional leader of the classroom. It was found that the teachers' influence on student learning remained despite the manner in which the teacher interacted with the class. A second consistency with the two studies involved teacher training. In both experiments, the teachers were subjected to a period of instruction on the methods of learning groups (Francisco, 2013) and change pedagogy (Kang, 2007). Student learning was positively impacted by the teachers who engaged in the learning groups as a guide toward new knowledge rather than an instructor. Students' learning was not impacted by the teachers who received instruction on change pedagogy; the researcher noted that the teachers were not using the full range of the students' cognitive resources (Kang, 2007). Teachers were also found to limit their own exposure to varying solution paths in mathematics, which limits student resources. During participation in a workshop, teachers were observed completing mathematical problems with holistic inconsistency and ignorance of one another's suggested methods. The ensuing discussion led to the conclusion that the teachers did not possess the conceptual understanding necessary to lead students beyond the limits of procedural learning (Liu & Thompson, 2004).

On the contrary, Kajander's 2007 study revealed that in-service grade seven teachers demonstrated an increase in mathematical knowledge for teaching as a result of professional development. More significantly, though, the teachers also developed an increased valuation of of conceptual understanding while decreasing the value they placed on procedural fluency in their classrooms. In a subsequent study, one teacher participant commented "there is a difference between teaching math and just math knowledge … you have to establish what the big ideas are for students" (In Kajander, 2010, 92). The learning culture is a formation of both the knowledge possessed by the teacher and the transference of that knowledge to the students. A second teacher noted "you have to understand the big idea first, and also be able to express an idea or concept [to students]" (In Kajander, 2010, 92).

Despite teachers' own perception of understanding, mathematical content instruction varies according to the particular concepts being taught. Instructional strategies for geometry can differ significantly from those used in calculus. For this reason, several researchers have provided insight into the relationship between concept and understanding. Consideration of how attitude influences conceptual understanding has revealed these two Geometry specific ideas: a) geometry increases student motivation to learn in grades three through six and b) successful teaching methods emphasize the use of tangible mathematical tools for geometric activities (Daher & Jaber, 2010). Student engagement in learning through tangible exercises increases their interest level, which can translate into increased understanding. The tangibility of Geometry, a study in shape and size, is more apparent than it is for Algebra. The skills that are taught in algebra based courses transcend many more years of mathematics studies for students' elementary and secondary schooling. A consistent model of emphasis on number competence in the early elementary grade levels can result in increased math skill in the later grade levels (Edens & Potter, 2012; Tseng et al., 2011). These math skills are the emphasis with which the author's study will proceed; that is, an emphasis on Algebra. According to the National Mathematics Advisory Panel (NMAP), Algebra curriculum should reflect the learning constructs of conceptual understanding, computational fluency, and problem solving skills (2008). The challenge with blending these three ingredients is deciding the relative quantity of each in a daily lesson or well-planned unit. NMAP (2008) encourages teachers to "emphasize these interrelations; taken together, conceptual understanding of mathematical operations, fluent execution of procedures, and fast access to number combinations jointly support effective and efficient problem solving." Skills in Algebra are preceded by skills in numeracy, and therefore the research regarding the development of both algorithmic and heuristic skills at the elementary level are critical to this current investigation.

Emphasis on mastery of numeracy is fundamental for success in higher level mathematics (Edens & Potter, 2012). A focus on elementary mathematics skills can lead to more progressive development of complex skills as students enter middle school and high school. Edens and Potter (2012) corroborate previous research confirming that increased counting skills are found in children who demonstrate spontaneous focusing on numerosity or SFON. The increased spontaneity of the most basic symbols in mathematics suggest a deep understanding of the foundational skills. Additionally, students demonstrating greater conceptual understanding have been found to show an affinity toward STEM (Science, Technology, Engineering and Mathemetics) related activities such as: creative social play, artistic design, technology utilization and block construction (Edens & Potter, 2012). These activities are consistent with the variety of theories and activities previously identified; namely, Lev Vygotsky's social constructivist theory and Seymour Papert's constructionist learning activities to enhance conceptual learning in mathematics (Edens & Potter, 2012; Francisco, 2013; Tseng et al., 2011).

Theme 2: Conceptually Based Learning Theories

The National Research Council (NRC) has called for reform in mathematics education moving from the instructionist concept of teaching to the constructivist theory of learning utilizing more problem-based discovery (Moldavan, 2008; Thornburg, 2013). Empirical evidence provides insights on conceptual understanding, symbol manipulation skills and problem-solving skills (Cai et al., 2013; Schoenfeld, 2013, 2014). Rather than move directly from the traditional delivery of curricular content, a new instructional strategy called problem-posing has been suggested where students are assessed on their understanding of new knowledge based on the level of problems that they construct. Cai et al. (2013) have found that the quality of the problems posed by the students was an indicator of their ability to solve problems posed by the teacher. The idea of problem posing raises an awareness of a skill that is often neglected in mathematics classrooms. Considering the algorithmic nature of mathematics content, creativity is infrequently a learning objective, but the strategy of problem-posing suggests a link between conceptual understanding to student creativity (Cai et al., 2013). The author's study was intended to uncover teachers' valuation of conceptual understanding, and by association, the valuation of creativity in the mathematics classroom. According to Star (2005), procedural fluency that is supported by some of the tenets of creativity: comprehension, flexibility and critical judgment, allows for conceptual understanding.

The investigation into the impact of STEM related activities that are grounded in the theories of experiential learning, constructionism and situated cognition can be obscured by the method of assessment used to determine the level of learning that has occurred. In a study designed to measure the impact of the use of iPads on student achievement, Carr (2012) reports student performance only on standardized tests rather than performance based assessments. The study demonstrates an attempt to utilize 21st century equipment for instructional purposes without the consistency of assessing 21st century learning due to the nature of standardized testing. Careful and consistent instructional strategies, practice situations and assessment methods all need to be considered when investigating the relationship between STEM and conceptual learning (Kabapinar, 2005; NMAP, 2008; Ernst & Clark, 2012).

Game design, concept cartoons and discovery learning are three examples of intervention strategies used under the umbrella of STEM activities that have been found to influence student learning. Unfortunately, the research is inconsistent in its conclusions. Although game design has been found to increase student motivation, the impact on conceptual learning remains inconclusive (Ernst & Clark, 2012). When students are faced with the challenge of demonstrating their learning through new and unique assessment methods, several factors can alter the outcomes. A sudden change from basic algorithmic responses can be intimidating for students, and result in a loss of self-confidence. Conversely, proper design of an assessment tool can lead students to greater understanding of material that had previously been too challenging

(Kabapinar, 2005). Research has shown that the implementation of such assessments and strategies are largely dependent on the teacher interactions with the students (Kabapinar, 2005).

As with other research, the direct and indirect influence of the teacher on student learning is a critical component to be addressed (Kabapinar, 2005). In a study examining discovery learning strategies in daily lessons rather than expository learning strategies, greater improvement among the students was realized as a result of the challenge of more difficult material. The material itself was not necessarily more challenging, but the acquisition of it was student driven rather than teacher driven. This subtle change produced significant positive results (Mandrin & Preckel, nd). The study conducted by Mandrin and Preckel was unique its cross-curricular learning environment. The discovery activities required students to engage in lessons relating math to science and vice-versa. This integration of content was also found in an investigation of L. Dee Fink's integrated course design (ICD) model that shifts the focus from rote knowledge to the application of skills toward the development of self-directed learners (Nicoll-Senft, 2009). Considering real world problems cannot usually be segregated into content specific categories, the ICD model is consistent with the Mandrin and Preckel experiment. The study of ICD corroborates other studies' successful implementation of problem-based learning as a strategy to show significant change in the students' application and integration of foundational knowledge (Nicoll-Senft, 2009).

Strategies such as Fink's ICD advocate a constructionist philosophy of education, and use that philosophy to promote the use of design challenges to teach math, science, reading, writing and engineering (Rogers & Portsmore, 2004). The concepts of hands-on learning, problem-solving and creative-thinking are all components of the relationship between conceptual understanding and procedural fluency. In the absence of this constructionist approach, students learn algorithms as an abstract set of procedures that can be applied to a finite set of circumstances (Boaler, 2000). The author examined how teachers' valuation of conceptual understanding and procedural fluency influences their pedagogical decisions to incorporate a more constructionist approach as a means to move students away from the abstract toward more concrete mathematical models. In conjunction with the constructionist approach, the amalgamation of conceptual understanding and procedural fluency is thought to be a product of situations in which students learn. These situations are proffered by teachers through artificial, yet authentic, problem-solving or creative-thinking scenarios. Situated learning contains an element of social interaction which allows students to build their knowledge and a deep understanding through their own experiences and the experiences of others (Hoyles, 1992; Boaler, 2000).

Theme 3: The Nature of Mathematics Learning

Mathematics is a dynamic discipline to be explored and created, rather than one that is discovered. Through a review of teacher interviews, Kaci Allen reports that routine math lessons are still being delivered to students despite teachers' self-reflections of constructivist and innovative strategies. The integration of STEM activities in mathematics classroom may be the basis for instructional reform to lead to deeper conceptual learning. As educators attempt to create a distinct connection between procedural learning and conceptual learning, the standards presented by NCTM (National Council of Teachers of Mathematics) 2000 and the Common Core Math Standards need to remain part of the equation (Allen). In a study examining student understanding of fractions, Alon (2012) reveals that students are only being taught the algorithms, and therefore, student learning is not being optimized. In this context, the very beginning levels of the NCTM math practices are not being realized in elementary classrooms.

Through direct intervention using a new conceptual teaching approach, Alon (2012) was able to demonstrate a positive correlation between conceptual teaching and student learning when compared to a traditional approach. Despite the suggestion of improvement in student learning, the study does not delineate the impact on student achievement from the use of the experimental conceptual method versus the enhancement of the teachers' instructional ability through training (Francisco, 2013; Kang, 2007; Liu & Thomspon, 2004). This ambiguity suggests that teacher training remains an important ingredient into the investigation of improved conceptual understanding in the wake of a focus on procedural teaching.

To further complicate the issue of conceptual understanding and procedural fluency, research has shown that students can acquire understanding on each of the two levels in isolation. Although the two-tiered acquisition has been demonstrated, research suggests that instructional methods may hamper the union of the two, and students are left in disequilibrium in terms of their holistic understanding (Arslan, 2010). The research is inconclusive regarding the sequence of learning from either algorithmic-to-heuristic approach or its reciprocal (Bednar & Sweeder, 2010; Byrnes & Wasik, 1991). The author's study sought to aid in providing greater definition to the existing sequential ambiguity by inquiring about teachers' values relative to procedural fluency and conceptual understanding, and the source of those values.

In a shift from the discussion of procedural fluency and conceptual understanding, a slight linguistic alteration provides some insight into the virtues of the progressive relationship between the two. Canobi (2008) found that conceptual relations help children to extend their procedural fluency beyond the particular problems they have already solved to new problems. Also, children can develop their reportable conceptual knowledge as a result of procedural experience. This change from procedural learning to procedural experience suggests the importance of constructivism, constructionism and situated learning (Boaler, 2000). The consideration of a student's experiences in learning may reveal an alternate path in the development of conceptual understanding. Students own examination of the learning, or the development of meta-cognitive skills, presents new information filling the gap between procedural fluency and conceptual understanding (Gogus, nd.).

Gaps in student learning seem to be the focus of the report from the NRC (Moldavan, 2008). In an attempt to address those gaps, teachers may need to develop conceptual teaching strategies that allow students to attach new schemata to existing experiences moving the acquired knowledge from short term memory to their working memory so that generalizable associations can be made (Gray et al., 1999). High achieving students were found to focus on flexible procedural understanding of a concept, which allowed them to manipulate their conceptual understanding in a predictable real world event. If students acquire the algorithm to a problem without further development of that algorithm, it remains fixed to one problem in isolation; authentic learning is not achieved (Gray et al., 1999). Furthermore, learning that is experienced in the company of others is enhanced and the participants become attached to the experience. Students, in turn, relate algorithms to the learning group. The learning group can collaborate on a concept with the intention of creating a shared meaning or construct of the holistic learning experience (Johnson & Galluzzo, nd).

In a report submitted by David Tall (2008), three phases of mathematical understanding are identified as sequentially critical to a student's mathematics lesson experience. The three phases are conceptual-embodied, proceptual-symbolic and axiomatic-formal. Tall suggests the necessity of an authentic understanding of a concept as a result of deliberate practice of learned processes and manipulation of mathematical symbols. Through implicit references, the author builds upon theories of social constructivism and situated learning, as well as, explicit acknowledgement of Ed Dubinsky's APOS theory consisting of action, process, object and schema (Tall, 2008). This study highlights the Van Hiele model of structure and insight relative to mathematical problem solving, and relates the model to conceptual understanding on three different levels. Teachers who unify the idea of the three phases suggested by Tall with the social learning experience studied by Johnson and Galluzzo could afford their students the opportunity to blend the algorithmic component to mathematics education with the heuristic component. The successful amalgamation of these concepts may be the answer to the NRC's desire to move away from the teaching of isolated skills and procedures to allow for emphasis on problem solving and sense-making (White-Fredette, 2009). The author identified the goals of the NRC as paramount to the significance of the proposed study in the progressive paradigm shift of the nature of mathematics as a content area of creativity and invention rather than one of discovery and chance.

Summary

This chapter examined both the theoretical and practical elements of mathematics education. Although the literature is rich with data, theories and intervention strategies, there are areas that have not yet been reported on. Most glaringly is an examination of the constructs of conceptual understanding and procedural fluency in conjunction with activities consistent with constructionism and situated learning while regarding teachers' values and beliefs. More specifically, each theme explored has research isolated in its use of either grade level, mathematical content or intervention strategies. Previous research that is focused on the impact that new intervention strategies has on mathematical education has not examined the theories of Seymour Papert's constructionism or Jean Lave's situated learning. Research has suggested that teachers' beliefs impact their approach to teaching (Hoyles, 1992; Boaler, 2000; Ambrose, 2004; Kajander, 2007; Kajander, 2010). The author of the study being reported here examined teachers' beliefs relative to procedural fluency and conceptual understanding in mathematics; how those beliefs were formed; and, the influence those beliefs have on their teaching.

Research has been found to recognize that teachers' math anxiety and math teaching anxiety have an adverse effect on student learning, as well as, students' attitude toward mathematics. This research is generally limited to elementary teachers, and does not provide information regarding student attitudes as they progress from the elementary grades. Additionally, research on a variety of intervention strategies has revealed some positive significance of technologically-driven activities in student achievement on math education. This research has generally been limited to secondary and post-secondary students. Additionally, the activities are typically electronic in nature. Finally, researchers have addressed the issues of procedural learning and conceptual understanding, both in isolation and in conjunction. The literature spans a wide range of mathematical skills and grade levels. Previous research raises several philosophical issues surrounding the interrelationship of the two learning constructs. Despite the vastness of this research, the literature, at times, is limited in the connections formed with previous educational psychologies.

The literature has offered a framework within which this study can proceed. Previous investigations have been synthesized around themes that are consistent with the objectives of the researcher regarding the relationship between teachers' values and beliefs regarding effective mathematics instruction and the various learning constructs and instructional strategies. In Chapter 3, the researcher outlined the methodology followed for completion of the study. The following chapter is organized to include the research design and rationale, descriptions of the site and population, the data collection and analysis procedures and a summary of the ethical considerations relative to the participants and associated organizations.

Chapter 3: Research Methodology

Introduction

The purpose of this two-phase, sequential explanatory mixed methods study was to investigate teachers' perspectives on procedural and conceptual learning strategies regarding their pedagogical value in secondary mathematics. In the first phase, a quantitative research question addressed the relationship of teachers' valuation of procedural fluency and conceptual understanding. Information from this initial phase was explored further in a second qualitative phase. In the second phase, the researcher conducted a focus group to gather additional information with a randomly selected group of participating teachers from a cross-section of the participating school districts. This second phase was intended to allow the researcher to explore the teachers' valuation of the two learning constructs with greater depth, and explained the quantitative data with significant clarity. The participants were teachers from various middle schools and high schools representing grades seven through twelve in public school districts in southeastern Pennsylvania. The study consisted of a snowball sampling scheme in order to collect appropriate and meaningful data from a wide array of participants. Snowball sampling "is a well-known purposive sampling technique that involves using informants or participants to identify additional cases who may be included in the study" (Teddlie & Tashakkori, 2009, p.175).

The methodology that follows will identify the site and population of the participants for the study, the research design and rationale, the specific details of the research methods and the ethical considerations that the researcher will safeguard throughout the duration of the study. The research methodology that follows throughout this chapter was designed to answer the following central research question: how do secondary mathematics teachers value procedural fluency and conceptual understanding in their instructional practices? The data from the first phase of this mixed methods study was collected using a modified form of a previously established instrument, the Perceptions of Math (POM) survey, in order to explore this overarching question (Kajander, 2007). The intention of the focus group in phase two of the study was to further explore the phenomenon of the value and perceptions of the two learning constructs by addressing the following two sub-questions:

1) How do secondary mathematics teachers' valuation of procedural fluency and conceptual understanding influence their pedagogical decisions?

2) How do teachers develop their value-system of procedural fluency and conceptual understanding in their instructional practices?

Both the National Council of Teachers of Mathematics (NCTM) and the National Research Council (NRC) have published documentation promoting the utilization of teaching strategies that result in a composite of procedural fluency with authentic conceptualization of numeracy and spatial relations (Kilpatrick, 2001; Moldavan, 2008; NCTM, 2000; NMAP, 2008). The continuum of mathematics education in K-12 public education resides in the presumed consistency of teaching among the elementary and secondary school ranks. In order to understand the degree of attention given to the various teaching foci, the researcher intended to study teachers' perspectives on the first two elements of the five strands of mathematical proficiency: procedural fluency, conceptual understanding, adaptive reasoning, strategic competence, and productive disposition.

Although the researcher acknowledges the need to be inclusive of K-12 education, this research study examined viewpoints of teachers currently teaching mathematics to students in secondary schools only; secondary schools according to the Pennsylvania Department of

Education includes grades seven through twelve (PDE, nd). An initial group of participating teachers were asked to complete a web-based survey for the purpose of quantitative data collection. The survey consisted of questions pertaining to demographics, experience, and educational background. Additionally, the survey asked participants to indicate their valuation of procedural fluency, conceptual understanding, and a blend of the two within the context of Algebraic skills. From the initial group of participants, a smaller sample size of teachers were selected to participate in a focus group intended to discuss the relationship between their valuation of the two learning constructs and the influence of their valuation on their instructional practices. A second outcome of the focus group was to describe the rationale for the teachers' current value system. This chapter begins with the methods employed for the study followed by descriptions of both the population and the sites used for the study, and concludes with the specific research methods utilized and a summary of the protection of participants' rights.

Research Design and Rationale

This research study was a sequential explanatory mixed-methods examination of secondary school teachers' perceptions regarding algorithmic and heuristic learning opportunities for mathematics students relative to Algebra related content. A sequential explanatory mixed-methods strategy "is characterized by the collection and analysis of quantitative data in a first phase of research followed by the collection and analysis of qualitative data in a second phase that builds on the results of the initial quantitative results" (Cresswell, p. 211, 2009). This strategy is depicted in Figure 3-1 below. The combination of data collected from both phases one and two of the study is aimed to yield the following: a) secondary mathematics teachers' valuation of procedural fluency and conceptual understanding, b) how teachers' valuation informs instruction, and c) the factors that have lead to the development of the self-reported valuations.

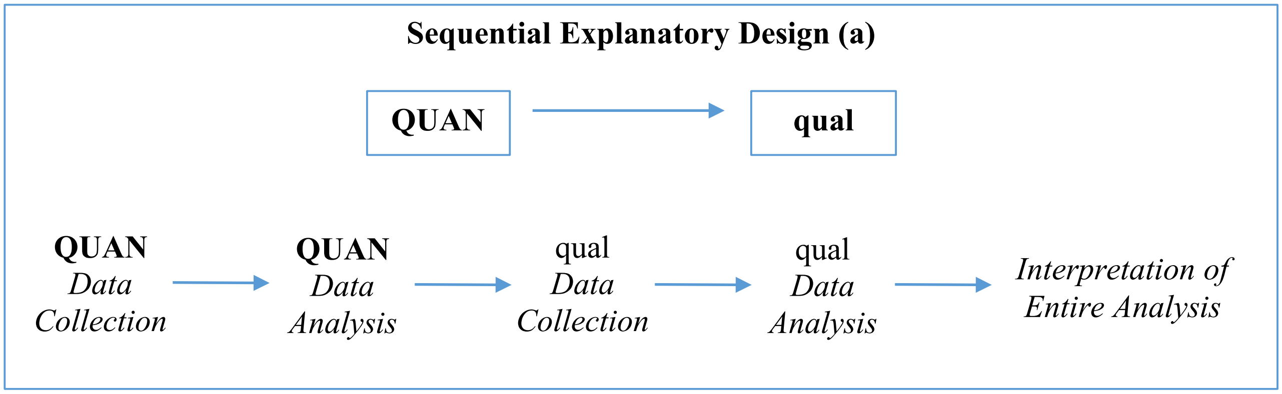

Figure 3-1: Sequential Explanatory Design from Cresswell (2009).

Two major outcomes anticipated from this study regarding the procedural and conceptual mathematical experiences for students were: a) what do teachers think and b) how does a teachers' thinking influence their actions. The discovery of what teachers think regarding the phenomenon of learning constructs was achieved by investigating the relationship between teachers' pedagogical decisions and their interpretation of the value of incorporating both conceptual understanding and procedural fluency in instruction, activities and assessments. A determination of how teachers act resulted from investigating the relationship between teachers' valuation of procedural fluency and conceptual understanding, and how their valuation influences instructional practices. In order to achieve these results, the researcher examined the viewpoints of public school teachers who had experience teaching courses based on Algebraic content in secondary schools situated in suburban southeastern Pennsylvania.

This mixed-methods approach afforded the researcher an opportunity to collect and analyze quantitative and qualitative data from the participants that resulted in a synthesis of descriptive, philosophical and explicatory knowledge relating the teachers' perspectives of mathematics as both a conceptual discipline, as well as, an algorithmic course of study

(Cresswell, 2009). Although the acquired data resulted from a quantitative instrument and a qualitative focus group, the researcher's approach to the study persisted through a phenomenological lens. The resultant data was based on participating teachers' experiences and philosophies of education. "The empirical phenomenological approach involves a return to experience in order to obtain comprehensive descriptions that provide the basis for a reflective structural analysis that portrays the essences of the experience" (Moustakas, 1994, p.12). The research design was grounded in the participants' values and beliefs. Using an explanatory mixed-methods design, the researcher was able to apply the theoretical perspective of the phenomenon being examined to the analysis of the quantitative data while utilizing the results of the qualitative phase to first inform and then further interpret the initial survey data (Cresswell, 2009). A detailed account of the participants and research sites is presented in the next section, followed by a complete description of the survey instrument and focus group protocol.

Site and Population

Population Description

The participants involved in this research study was a collection of secondary school teachers currently practicing in traditional public school districts across various suburban areas of southeastern Pennsylvania. The target population of teachers have earned a Pennsylvania Instructional II teaching certificate in the area mathematics, and have been practicing teachers in their certified area of mathematics for at least four years. The population of secondary mathematics teachers was represented by a sample of approximately 100 teachers selected through an opportunistic snowball sampling protocol. Participants included those who had been actively teaching mathematics content associated with standards consistent with Algebra since at least 2013. The most recent change to the mathematics standards in Pennsylvania occurred in

2014, and the phenomenon being studied relies on the discrepant practices that may have developed as a result of the changes.

Site Description

The participating teachers were gathered from various school districts located in suburban areas of southeastern Pennsylvania. The selected schools were traditional public schools, and were selected based on a convenience sampling. The geographic area that constitutes the researcher's target population included three counties consisting of a total of 44 school districts. The researcher's personal and professional network of teachers and administrators was utilized to gain access to seven of the 44 school districts. Rather than selecting a small sample of schools from the chosen geographic area, the researcher gathered data from representative schools within each of three different counties to increase the efforts of generalizing the resultant data and findings of the study.

Although a complete numerical description of the school districts can be found in Appendix A, the researcher provides some description of sample school districts as exemplars of the entire population. The largest school district in the study has eight different secondary schools consisting of approximately 5,500 students attending the middle schools and nearly 5,000 students attending the high schools. In order to accommodate the estimated 10,500 students, the combined faculty eclipses 700 teachers. Considering this particular study involves the beliefs of mathematics teachers, approximately 12% of the district faculty are potentially eligible participants for the study. The smallest school district in the study has just two secondary schools consisting of nearly 1,200 students and more than 120 teachers. Similar to the larger school district, an estimated 12% of the district faculty are potentially eligible participants for the study. Each of the targeted school districts are located within a 70-mile radius from a major metropolitan in southeastern Pennsylvania covering three suburban counties. The participating school districts were selected because of personal and professional relationships that had been previously established by the researcher.

Site Access

The schools that were included in the research study were selected based on a convenience sampling protocol. Considering the nature of convenience sampling and the previously established personal and professional relationships with the district administrators and teachers, the researcher did not experience any issues of access. This mixed-methods study examined the perceptions of mathematics teachers in the secondary schools. The researcher acknowledged that these perceptions could have been influenced by district and/or building cultures, but teacher affiliations with specific schools or school districts was not being studied nor reported. The data analysis procedures do not include any attempt to correlate specific teachers' or teacher cohorts' survey responses or focus group sentiments with particular school buildings or school districts. Data collected pertaining to the names of school buildings or school districts was for descriptive and phase two participant selection purposes only.

Research Methods

Description of Methods Used

This explanatory mixed-methods research study utilized a survey instrument and a focus group protocol for data collection purposes. The survey instrument was not an original design by the researcher, but rather a modified instrument generated at Lakehead University titled the Perceptions of Math (POM). The original POM has two components, but the purpose of this study required the use of just one of those elements. The POM has both a practical element that measures conceptual and procedural knowledge, and a value element that measures participants' valuation of procedural fluency and conceptual understanding. The original two-part survey was intended to assess "both preservice teacher knowledge and beliefs about mathematics" (Kajander, 2007). The survey was first introduced and utilized in a study in 2005, but was revised following an item analysis that revealed both a restructuring of some items, and deletion of others. Validation of the new instrument ensued, and is now being presented as the instrument for this current study (Kajander, 2007). A detailed description of the POM is included later in this section. The validity and reliability of the survey are provided in this description of the measurement tool as part of the 2007 study by Kajander. This study on teachers' valuation of procedural fluency and conceptual understanding does not involve a knowledge component, and therefore, the first element of the POM was not used.

The essence of the instrument remained in its original form with the exception of the addition of demographic questions designed to identify the participants according to present teaching assignment, teaching experience, educational background and certification(s), and information regarding teachers' experience with pedagogy directly related to mathematical concepts regarding to Algebra content. Other revisions to the study included the addition of questions and a clarification of the object of some survey statements such that the participant considers the student as the object, rather than the teacher as the object. For example, the second statement on the original survey reads "it is important to me to really understand how and why math procedures work" (Kajander, 2007). The revised statement is "it is important to me that my students really understand how and why math procedures work". The original survey can be found in Appendix B, while the modified survey is located in Appendix C. Authorization for the utilization of the survey and permission to revise the survey were acquired through both electronic communication and telecommunication with Dr. Ann Kajander at Lakehead University. Documentation of this consent can be found in Appendix D.

The POM consists of 20 statements. Ten of the statements emphasize procedural fluency, while the other half emphasize conceptual understanding. The survey instructs participants to respond to each statement using a Likert-type scale of 0, 1, 2 or 3. A selection of 0 indicates the participant disagrees with the statement, and a score of 3 indicates the participants' agreement with the statement. Validity of the instrument is represented in the context of the 2007 study conducted by Kajander, "in a parallel study with in-service teachers, the survey was subsequently administered to a sample of in-service grade 7 teachers along with other standard measures of teacher knowledge (Hill et al., 2005) and beliefs (Ross et al., 2003) providing initial validation for the new instrument with in-service teachers (see Kajander et al., 2006; Kajander & Zerpa, 2006)". Additionally, the reliability evidence is also contained within the 2007 study, and was achieved through the use of a pre-test and post-test analysis. Kajander reports that the reliability of the initial survey (prior to her own revisions) using Chronbach's alpha were established at 0.78 for the Procedural Values and 0.70 for the Conceptual Values. After the revisions were made, and the testing was repeated the reliability remained the same for Procedural values, but increased to 0.82 for Conceptual Values.

The survey instrument utilized for this study consists of 27 statements, and participants responded to each statement using a Likert-type scale of 1, 2, 3, 4 or 5. The participant indicated their level of agreement or disagreement with each statement by choosing the most appropriate number the corresponds as follows: 1 – disagree, 2 – somewhat disagree, 3 – neither agree nor disagree, 4 – somewhat agree and 5 – agree. Although the existing POM survey addresses the sentiment of the researcher, the categorization of statements according to the two learning constructs has been expanded to include a third category representing the blending of procedural fluency and conceptual understanding, or learning synergy. Additionally, the statements on the new 27-item survey specifically address these three components of teaching: instruction, activity design and assessment. The various statements can be used to identify how a participant values each of these three components in each of the three categories. The researcher completed a table of specifications to ensure that each type of statement addressed an identifiable component of teaching within an identifiable construct. The table of specifications also allowed the researcher to analyze the survey to avoid an over-emphasis on one construct-component combination while also ensuring an even distribution of statements across all construct-component combinations.

Subsequent to the collection and analysis of the quantitative data from the Beliefs and Values for Teaching Mathematics (BVTM) Survey, the researcher continued the study by investigating how teachers' valuation of procedural fluency and conceptual understanding influence their instructional practices, as well as, the factors that they identify as having developed their values. Focus group participants were randomly selected from those teachers that completed the survey in phase one. The utilization of a focus group allowed the researcher to gather data that was "deeper and richer" because of the social interaction compared to data obtained from one-on-one interviews (Rabiee, 2004, p. 656). The focus group protocol can be found in Appendix E. Although the participants' contribution to the study was based on self-reporting the foundation for their value system, as well as, the influence their values have on their instructional practices, the focus group method required group interaction "encouraging more honest and spontaneous expression of views and a wider range of responses" (Rabiee, 2004, p. 656). This form of qualitative research is designed to help explain a situation rather than uncover a definitive truth. This study attempted to ascertain specific philosophies that form teachers' approach to instruction; the focus group protocol was designed to "promote self-disclosure among participants" so that the researcher could report on how they really think and feel (Krueger & Casey, 2014, p.10).

The focus group protocol, which can be found in Appendix E, was adopted from an instrument utilized for a presentation to the Association of Mathematics Teacher Educators (AMTE) in 2005. The survey, titled the AMTE Voluntary Survey, consists of a demographics component followed by four distinct parts inquiring about curriculum and instruction, student understanding, assessment and change in practice (Bahr and Bossé, 2008). Aside from the demographics questions, the instrument includes 31 questions consisting of an assortment of open response and Likert-type scale items. For the purpose of this current study, the researcher isolated questions from the AMTE survey that pertain particularly to the investigation relative to procedural fluency and conceptual understanding and will elicit an open discussion for the focus group. Permission to utilize and modify the AMTE survey was obtained through electronic communication with the author of the study. Documentation of this consent can be found in Appendix G.

Data Analysis Procedures

The above data collection methods were designed to retrieve information from participating teachers so that the following research questions could be answered.

Central question: how do secondary mathematics teachers value procedural fluency and conceptual understanding in their instructional practices?

Sub-question 1: how do secondary mathematics teachers' valuation of procedural fluency and conceptual understanding influence their pedagogical decisions?

Sub-question 2: how do teachers develop their value-system of procedural fluency and conceptual understanding in their instructional practices?

The central question was addressed utilizing the data collected from the survey instrument, and the two sub-questions were answered from the rich information provided by the participants in the focus group. A more detailed account of the variables being studied, the research questions and the data collection instruments can be found in Table 3-1. Although the research questions are constructed with distinct variables, the survey instrument and focus group protocol were designed to yield overlapping data points. The table is organized in the same sequence that the research questions were originally presented, and sequentially consistent with the nature of the researcher's approach of sequential explanatory mixed-methods.

Table 3-1: Variables, Research Questions and Data Collection Instruments

<table>
<tr><th>Variable</th><th>Research Question</th><th>Items on Survey</th><th>Items on Focus Group Protocol</th></tr>
<tr><td>Valuation of procedural fluency</td><td rowspan="3">How do secondary mathematics teachers value procedural fluency and conceptual understanding in their instructional practices in southeastern Pennsylvania?</td><td>2, 5, 6, 8, 11, 14, 16, 19, and 23</td><td>1, 3</td></tr>
<tr><td>Valuation of conceptual understanding</td><td>1, 7, 10, 15, 18, 21, 24, 25, and 27</td><td>2, 3</td></tr>
<tr><td>Valuation of a blend of procedural fluency and conceptual understanding</td><td>3, 4, 9, 12, 13, 17, 22, 26, and 20</td><td>1, 2, 3</td></tr>
<tr><td>Influence of valuations on instructional practices</td><td>How do secondary mathematics teachers' valuation of procedural fluency and conceptual understanding influence their pedagogical decisions?</td><td></td><td>1, 2, 3, 4 and 5</td></tr>
<tr><td>Factors that lead to the development of teacher valuations</td><td>How do teachers develop their value-system of procedural fluency and conceptual understanding in their instructional practices?</td><td></td><td>1, 2, 3, 4, and 5</td></tr>
</table>

Utilizing a mixed-methods approach resulted in the collection of quantitative data from the survey instrument and qualitative information from the focus group. As the data collection methods are different for each of the two phases of the study, so too did the data analysis consist of two phases. The quantitative data was analyzed using SPSS software in order to prepare the descriptive statistics that summarize teachers' valuation of procedural fluency and conceptual understanding. The qualitative data was analyzed using NVivo software in order to collect, code, and classify according to themes the statements recorded from the focus group. A more detailed description of the data analysis is provided in the following two sections.

Quantitative Analysis

This research study consisted of a first phase of administering a survey instrument to secondary mathematics teachers to collect data pertaining to their valuation of procedural fluency, conceptual understanding, and learning synergy. The survey consisted of 27-statements and a Likert-type scale for participants to respond to each statement on a scale of one-to-five. The numeric scale is associated with participants' degree of agreement with each statement; the scale is graduated in five increments from "disagree" represented by a score of one to "agree" represented by a score of five.

The data was entered into the SPSS software with data columns consisting of the participant followed by the response for each statement. Descriptive statistics were computed in order to indicate the mean, the standard deviation, and the range of responses to each statement. The data was then categorized according to each of the two learning constructs, with the calculations of mean, standard deviation and range repeated for each separate construct. These calculated scores were utilized to identify the propensity for each of the constructs, but more importantly depict the collective feelings of secondary mathematics teachers with regard to procedural fluency and conceptual understanding. The results of the descriptive statistics were reported in a table of results with both the overall scores, as well as, the stratified scores according to learning construct. This table will be utilized to answer the central question of this research study, and can be found in Chapter 4.

Qualitative Analysis

Following the first phase of the study, participants were randomly selected to participate in a focus group interview during phase two of the study. The focus group interview protocol consisted of five general questions to lead the discussion. With participants' permission, the session was both audio-recorded and video-recorded. The recording of the focus group interview allowed the researcher to accurately capture all of the responses, as well as, the associated respondents.

The focus group interview was approached from a phenomenological perspective. A phenomenological methodology involves consideration of the shared experience among the teacher participants in order to acquire an inclusive depiction of the phenomenon of teaching mathematics with the various learning constructs as a foundation for individual pedagogy. (Moustakas, 1994). This approach demands the researcher to bracket personal beliefs throughout the focus group interview in order to gain the unbiased data from the participants.

In order to delve deeper into the valuation of the two learning constructs, the researcher conducted a focus group interview consisting of a small group of teachers selected from the participants in phase one of the study. The data recordings were transcribed in preparation for analysis. The NVivo software program was utilized to manage, organize, and code the data. Coding of the data "fragments the interview into separate categories, forcing one to look at each detail" so that significant statements and themes can be identified as a means to addressing the research questions regarding the influence of teachers' valuation on instructional strategies and the factors that lead to the development of identified beliefs (Bloomberg & Volpe, 2008).

The resultant themes from the focus group data were synthesized with the categorical and collective descriptive statistics. This synthesis presented a more complete understanding of teachers' values, the impact of those values, and the rationale for those values. The researcher aimed to answer each of the three research questions through the presentation of the findings that are discovered through this analysis phase of the study.

Stages of Data Collection

This research study consisted of two-phases. Phase one involved a 27-question survey, and phase two consisted of a focus group. The participants were secondary mathematics teachers in various suburban school districts located in southeastern Pennsylvania. The study was scheduled to commence in the Spring of 2017. Data collection began with the distribution of a web-based survey for participating teachers to complete. The survey was administered through an email communication to potential participants. Teachers received an electronic invitation to participate in the survey. Participants were given an opportunity to read a description of the study including the purpose and intentions of the data being collected. Assurances of confidentiality were also addressed in conjunction with the distributed invitation. Secondary mathematics teachers were identified through the use of district website staffing, and contact information was acquired from the districts' on-line staff directories.

The targeted distribution of the survey was April, 2017. The survey was designed to be completed within a twenty-minute time period, and was meant to be submitted through an on-line response system. The survey was closed in June, 2017.

The researcher used the data from the survey to identify participants for phase 2 of the study. Survey responses were analyzed such that participants for the focus group represented a cross-section of the data collected. The focus group session was scheduled with consent and collaboration from the participating teachers with a targeted time frame of June, 2017. The selection of a location for the focus group was mutually convenient for all participants. Considering the wide geographic area of the study, some participants were invited to attend the focus group through an agreed upon electronic medium, rather than in-person; none of the participants exercised this option.

The results of this study were dependent on the honest and accurate reflections provided by the participants during this period of data collection. In order to provide assurances of anonymity, confidentiality and well-being, the following section outlines the ethical considerations for this research study.

Ethical Considerations

This study consisted of an examination of teachers' perspectives regarding mathematics. This study did not present any identifiable risks to the participants or the locations at which the study will take place. The study only commenced after it had been reviewed and approved by Drexel University through the IRB process. Each participant's involvement in this research study was preceded by either written or electronic consent, which described the potential risks of the study, the anticipated benefits of the study, and the explicit confidentiality that would be sustained by the researcher. The participants were provided the opportunity to review the research methodology prior to granting consent. Lastly, the participants were assured that their involvement in this research was completely anonymous, and that any use of identifying characteristics would be removed or altered in order to maintain their confidentiality.

This research study involved the participation of human subjects, therefore, specific consent to be included in the data collection was required. All of the participants were adults, and were made aware of any potential risks of the study. Additionally, the organizations that the participants represent were afforded the same full disclosure of the activities and associated risks with the study by association with the participants. The requisite ethical considerations were rendered with the intention of safeguarding participants from any situation deemed mentally or physically harmful. If the participating teacher developed any aversion to the activities of the study, he or she was free to withdraw without fear of consequence.

Chapter 4: Findings, Results, and Interpretations

Introduction

The purpose of this research study was to examine the consistency of an equivalent attribution of the two learning constructs prescribed by procedural fluency and conceptual understanding through the delivery of mathematics instruction in secondary public schools. This mixed methods investigation was conducted with a phenomenological lens, and directed by both a quantitative exploration and qualitative investigation of secondary mathematics teachers' perspectives practicing in grades seven through twelve across several suburban school districts in southeastern Pennsylvania (PDE, nd).

This study examined teachers' perspectives regarding the necessity of both procedural fluency and conceptual learning in courses emphasizing Algebraic skills. The research questions used in the study provided a quantitative depiction of teachers' valuation of the two learning constructs and a qualitative description of how and why teachers interpret the nature of the mandated learning constructs as they pertain to instructional delivery of Algebraic concepts. Through the use of a survey instrument, participants provided feedback on each construct individually, as well as, collectively and with reference to the necessity of learning synergy in secondary mathematics. In the second phase of the study, the researcher conducted a focus group interview consisting of a sample of the surveyed participants. The focus group was intended to further explore the beliefs and rationale for those beliefs that teachers possess relative to the topic of procedural fluency and conceptual understanding in secondary mathematics.

The researcher employed a sequential explanatory mixed methods approach to the design of this study. Phase one of the study consisted of a survey with questions that are categorized by each of the two learning constructs: a) procedural fluency and b) conceptual understanding, as well as, a blend of the two. The survey was designed to provide results in an attempt to answer the central research question below.

Central Question: How do secondary mathematics teachers value procedural fluency and conceptual understanding in their instructional practices?

Consistent with a sequential explanatory mixed-methods design, the analysis of phase one data was conducted prior to progressing to phase two (Cresswell, 2009). Phase two consisted of a focus group that included participants randomly selected from the data collected in phase one. The data and subsequent analysis resultant to the focus group interview was utilized to address the two sub-questions that follow:

Sub-question 1: How do secondary mathematics teachers' valuation of procedural fluency and conceptual understanding influence their pedagogical decisions?

Sub-question 2: How do teachers develop their value-system of procedural fluency and conceptual understanding in their instructional practices?

Findings

The pursuit of data to aid in formulating an answer to the Central Research Question regarding teachers' valuation of procedural fluency and conceptual understanding began with a survey instrument that was administered on-line. The survey was composed of both demographic questions and questions pertaining to pedagogical experience and philosophical mindsets related to the teaching and learning of mathematics. The survey instrument included 27 statements, and participants were asked to respond to each statement using a Likert-type scale of 1, 2, 3, 4 or 5. The participants indicated their level of agreement or disagreement with each statement by choosing the most appropriate number that corresponds as follows: 1 – disagree, 2 – somewhat disagree, 3 – neither agree or disagree, 4 – somewhat agree, and 5 – agree. The 27 statements were constructed to address specific conditions of three different constructs: conceptual understanding, procedural fluency, and a blend of these two strands of mathematical proficiency that the researcher refers to as learning synergy. Furthermore, the survey statements target three different categories critical to the nature of teaching: instruction, activity and assessment.

The first phase of this sequential mixed methods study included 58 voluntary participants. The participants represent twelve different school districts from five different counties in southeastern Pennsylvania. All of the participants indicated experience teaching mathematics to students within the range of levels from grade 7 to grade 12. The average length of the experience in teaching mathematics among the participants was 16 years, with the majority of participants still practicing at the time this study was conducted. The following paragraphs are a description of the findings collected from the participants in the quantitative phase of the study. The tables included in the narrative that follows indicate the percentage of participants that responded to each statement using a particular value on the Likert-type scale and the mean rating and standard deviation for each statement. The tables also depict the overall mean rating and standard deviation for the identified construct or category as a whole.

Quantitative Survey Data Organized by Construct

The quantitative phase of the study consisted of data collected from a survey. Nine of the 27 statements on the survey focused the participants' thinking on each of the three constructs: conceptual understanding, procedural fluency and learning synergy. The data was organized around these constructs to clarify teachers' valuation of each construct independently from the others and independently from the three categories (instruction, activity and assessment) reflected in the statements.

Conceptual Understanding

The nine statements that were intended to uncover teachers' valuation of conceptual understanding in mathematics were statements number 1, 7, 10, 15, 18, 21, 24, 25, and 27. The statements of conceptual understanding are characterized by ideas of understanding, student ownership, deduction, and construction; for example, statement 21 reads "[s]tudents should be able to construct formulas and/or procedures to solving problems in order to demonstrate their understanding". With Likert-scale ratings of 4 for "somewhat agree" and 5 for "agree", the participants' responses varied from a mean rating of 4.02 for statement 21 to 4.87 for statement 18, or a range of just 0.85. Overall, the mean rating for the nine statements was 4.58 with a standard deviation of 0.32. Eight of the nine statements resulted in zero participants responding with a rating of 1 or "Disagree". One participant disagreed with statement 15, which reads "most math activities and exercises should include a variety of assessment strategies, some of which are not necessarily predicated on finding the correct solution". Although one participant disagreed, zero participants rated this statement with a Likert-type scale value of 2 for "somewhat disagree". Two statements, numbers 1 and 18, found participants responding by either agreeing or somewhat agreeing with no responses less than a Likert-type scale rating of four. In summary, there is a strong general agreement with the overall construct of conceptual understanding.

A summary of the data for the statements related to conceptual understanding can be found in Table 4-1: Conceptual Understanding Construct – Summary of Survey Results.

Table 4-1: Conceptual Understanding Construct - Summary of Survey Results

Statement	% Disagree (1)	% Somewhat Disagree (2)	% Neither Agree nor Disagree (3)	% Somewhat Agree (4)	% Agree (5)	Mean (μ)	Standard Deviation (σ)
1	0.00	0.00	0.00	13.73	86.27	4.86	0.35
7	0.00	1.96	0.00	35.29	62.75	4.59	0.61
10	0.00	0.00	1.92	23.53	74.51	4.73	0.49
15	1.92	0.00	3.85	34.62	59.62	4.50	0.75
18	0.00	0.00	0.00	13.46	86.54	4.87	0.34
21	0.00	3.85	17.31	50.98	27.45	4.02	0.79
24	0.00	0.00	1.92	13.46	84.62	4.93	0.43
25	0.00	0.00	19.23	40.38	38.46	4.15	0.80
27	0.00	1.92	1.92	30.77	65.38	4.60	0.63
Mean (μ)	**4.58**						
Std. Dev. (σ)	**0.32**						

Procedural Fluency

Nine statements were intended to reveal teachers' valuation of procedural fluency in mathematics; these statements are numbers 2, 5, 6, 8, 11, 14, 16, 19, and 23. The statements on the survey that are identifiably associated with procedural fluency are indicated by key terms synonymous to steps, memory, procedural skill, and isolation. An example of a statement of procedural fluency is statement 19: "[i]n mathematics, students usually only need to learn to solve problems using one method". The participants' responses for this construct were found to be more variable as each rating on the Likert-type scale was utilized to some degree for each of these nine statements. The range of the mean rating responses from the participants was 1.82 as determined by the difference between 1.63 for statement 19 to 3.45 for statement 5. Only three of nine statements had a mean rating value greater than three, and the overall mean rating for all nine statements was 2.61 and a standard deviation of 0.62. In comparison to the results of the statements pertaining to conceptual understanding, the overall impression from the participants was one of disagreement or neutrality toward the statements regarding procedural fluency.

A summary of the data for the statements related to procedural fluency can be found in Table 4-2: Procedural Fluency Construct – Summary of Survey Results.

Table 4-2: Procedural Fluency Construct - Summary of Survey Results

Statement	% Disagree (1)	% Somewhat Disagree (2)	% Neither Agree nor Disagree (3)	% Somewhat Agree (4)	% Agree (5)	Mean (μ)	Standard Deviation (σ)
2	5.77	23.08	13.46	36.54	21.15	3.44	1.23
5	5.77	11.54	32.69	29.41	19.61	3.45	1.12
6	23.08	38.46	13.46	21.57	1.96	2.39	1.13
8	32.69	38.46	7.69	17.31	3.85	2.21	1.19
11	9.62	17.31	28.85	34.62	9.62	3.17	1.13
14	30.77	36.54	9.62	17.65	3.92	2.25	1.20
16	25.00	26.92	30.77	13.46	3.85	2.44	1.13
19	55.77	30.77	9.62	1.92	1.92	1.63	0.89
23	25.00	26.92	25.00	19.23	3.85	2.50	1.18
Mean (μ)	**2.61**						
Std. Dev. (σ)	**0.62**						

Learning Synergy

The remaining nine statements were designed to show teachers' valuation of a blend of conceptual understanding and procedural fluency in mathematics; these statements are number 3, 4, 9, 12, 13, 17, 20, 22, and 26. The learning synergy statements reflected themes around variability of approach, multiple forms of assessment, and a progression from simple to complex. An example of a statement reflecting the construct of learning synergy, statement 20 says "[s]tudents usually struggle to apply new procedures to unique problems even though they have already demonstrated an understanding of how to solve related example problems". Although the variability of the responses was the lowest compared to the other two constructs, the range was the greatest of all three constructs being measured. The minimum and maximum mean rating responses from the participants were 1.90 for statement 22 and 4.62 for statement 9, respectively, resulting in a range of 2.72. While the overall mean rating for all nine statements was 3.70 and a standard deviation of 0.22, four of the nine statements had a mean rating greater than 4.00. The survey results for the statements pertaining to learning synergy suggest greater agreement than the results regarding procedural fluency, but the degree of agreement is not quite as high as the results for the statements focused on conceptual understanding.

A summary of the data for the statements related to learning synergy can be found in Table 4-3: Learning Synergy Construct – Summary of Survey Results.

Table 4-3: Learning Synergy Construct - Summary of Survey Results

Statement	% Disagree (1)	% Somewhat Disagree (2)	% Neither Agree nor Disagree (3)	% Somewhat Agree (4)	% Agree (5)	Mean (μ)	Standard Deviation (σ)
3	7.69	5.77	23.08	32.69	30.77	3.73	1.19
4	0.00	1.92	5.77	23.08	69.23	4.60	0.69
9	0.00	0.00	7.69	23.08	69.23	4.62	0.63
12	3.85	25.00	36.54	19.61	13.73	3.14	1.08
13	3.85	7.69	11.54	27.45	49.02	4.10	1.14
17	1.92	3.85	9.62	51.92	32.69	4.10	0.87
20	1.92	7.69	15.38	52.00	22.00	3.84	0.93
22	36.54	44.23	7.69	9.80	0.00	1.90	0.92
26	7.69	25.00	21.15	25.00	21.15	3.27	1.27
Mean (μ)	**3.70**						
Std. Dev. (σ)	**0.22**						

Quantitative Survey Data Organized by Category

The previous set of tables and accompanying narrative was a presentation of the data that reflects teachers' valuation of the three constructs that are the focus of the Central Research Question for this study. The following summary data also addresses the Central Research Question, but the data is organized around the three categories of the survey statements independent of the constructs. These categories are general classifications of three critical pedagogical aspects of secondary mathematics: instruction, activity and assessment.

Instruction

The following nine statements refer to a teachers' use of instruction as it relates to conceptual understanding, procedural fluency and a blend of conceptual understanding and procedural fluency: 1, 2, 4, 5, 7, 11,17, 18, and 22. The mean rating response for all nine statements was 3.89 with a standard deviation of 0.33. The range of mean rating responses across all nine statements related to instruction was 2.96 with a minimum of 1.90 and a maximum of 4.86.

A summary of the data for the statements related to teachers' instructional practices can be found in Table 4-4: Instruction Category – Summary of Results.

Table 4-4: Instruction Category - Summary of Survey Results

Statement	% Disagree (1)	% Somewhat Disagree (2)	% Neither Agree nor Disagree (3)	% Somewhat Agree (4)	% Agree (5)	Mean (μ)	Standard Deviation (σ)
1	0.00	0.00	0.00	13.73	86.27	4.86	0.35
2	5.77	23.08	13.46	36.54	21.15	3.44	1.23
4	0.00	1.92	5.77	23.08	69.23	4.60	0.69
5	5.77	11.54	32.69	29.41	19.61	3.45	1.12
7	0.00	1.96	0.00	35.29	62.75	4.59	0.61
11	9.62	17.31	28.85	34.62	9.62	3.17	1.13
17	1.92	3.85	9.62	51.92	32.69	4.10	0.87
18	0.00	0.00	0.00	13.46	86.54	4.87	0.34
22	36.54	44.23	7.69	9.80	0.00	1.90	0.92
Mean (μ)	**3.89**						
Std. Dev. (σ)	**0.33**						

Activity

Nine statements were constructed to reflect teachers' use and design of activities within the three constructs being studied. These nine statements are 3, 8, 10, 12, 14, 16, 21, 24 and 26. The mean rating response of 3.41 and standard deviation of 0.32 of the activity category is similar to the values reported for the instruction category. The range of mean rating responses across all nine statements related to activity was 2.72 with a minimum of 2.21 and a maximum of 4.93.

A summary of the data for the statements related to teachers' use and design of activities can be found in Table 4-5: Activity Category – Summary of Results.

Table 4-5: Activity Category - Summary of Survey Results

Statement	% Disagree (1)	% Somewhat Disagree (2)	% Neither Agree nor Disagree (3)	% Somewhat Agree (4)	% Agree (5)	Mean (μ)	Standard Deviation (σ)
3	7.69	5.77	23.08	32.69	30.77	3.73	1.19
8	32.69	38.46	7.69	17.31	3.85	2.21	1.19
10	0.00	0.00	1.92	23.53	74.51	4.73	0.49
12	3.85	25.00	36.54	19.61	13.73	3.14	1.08
14	30.77	36.54	9.62	17.65	3.92	2.25	1.20
16	25.00	26.92	30.77	13.46	3.85	2.44	1.13
21	0.00	3.85	17.31	50.98	27.45	4.02	0.79
24	0.00	0.00	1.92	13.46	84.62	4.93	0.43
26	7.69	25.00	21.15	25.00	21.15	3.27	1.27
Mean (μ)	**3.41**						
Std. Dev. (σ)	**0.32**						

Assessment

The final category of assessment can be found in these nine statements: 6, 9, 13, 15, 19, 20, 23, 25, and 27. These statements were formulated to inquire about teachers' design and administration of assessments in each of the three constructs: conceptual understanding, procedural fluency, and learning synergy. The mean rating response of 3.59 represents an intermediary value among the three categories, while the standard deviation of 1.12 is the highest of all three categories. The range of mean rating responses across the nine statements in the assessment category is 2.99 with a minimum of 1.63 and a maximum of 4.62.

A summary of the data for the statements related to assessment design and administration can be found in Table 4-6: Assessment Category – Summary of Results.

Table 4-6: Assessment Category - Summary of Survey Results

Statement	% Disagree (1)	% Somewhat Disagree (2)	% Neither Agree nor Disagree (3)	% Somewhat Agree (4)	% Agree (5)	Mean (μ)	Standard Deviation (σ)
6	23.08	38.46	13.46	21.57	1.96	2.39	1.13
9	0.00	0.00	7.69	23.08	69.23	4.62	0.63
13	3.85	7.69	11.54	27.45	49.02	4.10	1.14
15	1.92	0.00	3.85	34.62	59.62	4.50	0.75
19	55.77	30.77	9.62	1.92	1.92	1.63	0.89
20	1.92	7.69	15.38	52.00	22.00	3.84	0.93
23	25.00	26.92	25.00	19.23	3.85	2.50	1.18
25	0.00	1.92	19.23	40.38	38.46	4.15	0.80
27	0.00	1.92	1.92	30.77	65.38	4.60	0.63
Mean (μ)	**3.59**						
Std. Dev. (σ)	**1.12**						

The subsequent section will present the findings of the focus group interview. A detailed discussion of the results and the relationship among the constructs and categories from the quantitative phase with the significant statements found in the qualitative phase of the study is provided in the final section of Chapter 4 titled Results and Interpretations.

Qualitative Focus Group Interview Data Organized by Question

Following the first phase of the study, participants were randomly selected to participate in a focus group interview based on their willingness as indicated at the conclusion of the survey. The focus group interview protocol consisted of five questions intended to discover qualitative explanations for the quantitative data collected in phase one. The participants were asked to discuss their philosophy, experiences and opinions on pedagogy of mathematics with respect to various aspects of mathematical proficiency in their training, their lessons and the curriculum from which they teach.

The first phase of the study consisted of 58 participants. In May 2017, eight of 58 participants were invited to participate in the focus group interview. Invitations were delivered electronically with a description of the interview process and purpose. Seven invitations were accepted, with the eighth potential participant withdrawing from consideration for phase two of the study due to a change in career. An additional invitation was issued to fill the vacancy in the focus group. The eight participants were contacted in order to agree on a date, time and location for the interview. The logistics of the session were agreed upon and an invitation to the session was delivered electronically. After a subsequent reminder was sent, one of the participants withdrew as a result of a conflict with the date that was not disclosed initially.

The focus group took place on June 15, 2017. Six of the seven invited participants were in attendance. The seventh participant did not show for the interview session unexpectedly. At the commencement of the session, each participant was presented with a copy of the approved consent document from the Drexel University IRB. The group was notified that the session would be audio and video recorded, and signed consent forms were received from each participant. The focus group consisted of teachers and administrators from five different public school districts in southeastern Pennsylvania. The teaching experience of the participants ranged from seven to 23 years. Four of the participants were assigned to a teaching position at the high school level at the time of the focus group interview session, while two participants had recently moved out of the classroom and into curriculum administration. The focus group began with the researcher presenting the purpose of the session followed by addressing the group with the first question. The session continued for approximately 65 minutes with each participant adding to the discussion in a conversational nature while answering a total of five questions as prescribed by the focus group interview protocol, which can be found in Appendix E. The following section of this document portrays the findings from the focus group interview. Any names that appear below have been changed from the names of the focus group participants in order to maintain anonymity and confidentiality.

Philosophy of Education

The focus group participants generally agreed that the teaching and learning of mathematics is rooted in the ability to perform the processes necessary to solve problems. Ray noted "if they don't know what they are doing, they can't really know the why behind it". Initially, there was a clear sequence to the teaching of math regarding procedural fluency and conceptual understanding that was pervasive around the discussion of math in generic terms. Lisa contributed "I think you need to take a leap of faith, follow the rules and see that it works and once it works and you continue to master that and then conceptual understanding comes". In this instance, mastery of the procedure is necessary prior to addressing the understanding of the concept.

This philosophy was highlighted when the discussion shifted to the more specific topic of students who struggle in math. The opinion that procedural learning takes precedence over conceptual understanding when students have a learning disability with regard to math does exist; some suggest that this is not the best way to teach math. Mary commented "those who struggle with math – we focus on the procedural aspects of the math which is a disservice to the students, an injustice to the students. We push them through with the procedural knowledge". In Mary's statement, the dichotomy of the philosophy around the teaching of mathematics is revealed. Most of the participants agreed that students need to have the procedural skill of performing math in order to be able to understand the concepts that lead to the derivation of the algorithm that governs the process. In some cases, the construct of procedural fluency becomes the sole focus of math instruction for students who have difficulty solving problems.

One teacher did oppose the general opinion of the group by noting that conceptual understanding should be the primary approach followed by the procedural fluency as the secondary tactic. Bill voiced the opinion that the "students need to be able to understand the concept before they can really follow procedures". While everyone agreed that both constructs exist and are necessary, the importance and method of inclusion of procedural fluency and conceptual understanding in their lessons did vary.

Promoting Procedural Fluency and/or Conceptual Understanding

Experience and authenticity were found to be the two most critical components to a lesson for teachers to incorporate procedural fluency and/or conceptual understanding. Sean related the following example:

> *For them to understand the surface area to volume ratio of a cell - it's a lot easier for them to understand it when we do the lab where they have three dimensional cubes with chemicals in them and they can see a literal color change and they can see which cube actually has the best surface area to volume ratio to allow for maximum movement of fluids in and out of the cell. So that's a lot different than saying let's just calculate surface area volume ratio which one is the best. So when they can actually do and see it that's when I've seen the best result and the most connection with the concepts.*

The action of performing an activity promotes both the learning of the concept and the practice of the procedural skill in a calculation. While Sean utilizes a relatable content area, such as science, to incorporate the two constructs of mathematical proficiency, Bill's approach is to gain student investment through interest. He says, "I try to start out with an activity they are interested in and draw them in before I go into all the procedures and steps".

Other participants expand on the theme of student interest, and also incorporate application with the approaches of experience and authenticity in their math lessons. In order to provide students with a purpose for the math they are learning, Ray says “when we get to the application piece they have the solid skill set and they can go through the process”. Although the group was in agreement with the notion that students’ experience with applying mathematical concepts promotes skill development, the need for a personal connection was paramount. The participants found that recognition of individual students’ needs and interests was a necessary first step in planning appropriate and differentiated activities that could address both procedural fluency and conceptual understanding. In continued agreement, the group felt that a definitive sequence from one to the other is an unrealistic and unnecessary consideration within the framework of mathematical proficiency.

The Natural Flow

There were two common themes revealed in the discussion of sequencing procedural fluency and conceptual understanding. The participants agreed that if a sequence did exist it would be variable and contingent on the students’ ability and background knowledge and the classroom dynamics. Each of these components contribute to the direction a teacher takes from lesson-to-lesson. Although teachers assemble a lesson plan around objectives with instructions, activities and assessments, feedback from the students can impact the flow of the predetermined lesson plan. Mary noted “it is really about the differentiation and the constant monitoring of the students just to have a pulse on how exactly they are perceiving and how they are as learners”. With differentiation, the goal remains the same “mastery of the conceptual understanding with the procedural skills and accuracy”. In order to achieve the goal, the teacher’s first task is to present the path to success with the understanding that the path may be interrupted at times.

Rather than a linear approach to incorporating the two constructs in a lesson, the participants expressed that the approach is continually reciprocating between conceptual and procedural. Bill stated "while you are doing the procedure you are also doing something to understand the concept. I don't think they can go separately and I don't think one goes ahead of the other". The focus group participants agreed on the distinct need to address both sides of the mathematical proficiency equation discussed in this study. Rather than a flow from one construct to the other, procedural fluency and conceptual understanding should be part of every lesson simultaneously or on-demand as a response to the students' needs. Matt recalled an experience during his time as a university student: "I had a professor who leaned towards conceptual and I needed procedural; I got frustrated. Having a teacher who responded to my needs made me grow as a math student". Despite the variability of the sequence between procedural fluency and conceptual understanding, the participants noted that there is a clear requirement for curriculum and resource materials to be aligned with the strands of mathematical proficiency.

The Alignment of Curriculum and Mathematical Proficiency

Most of the participants began the discussion around curriculum by citing experience with the Understanding By Design model with specific reference to the components: essential questions and enduring understandings. These components were associated with the application of mathematical concepts and performance tasks requiring the students to demonstrate their computational abilities while solving problems. Bill designs problem solving activities based on the essential questions in the curriculum "to see if they understand the whole picture, which I would say answers the whole conceptual understanding thing". As a means to assess the students' progress toward the enduring understandings, participation in both formative assessments and summative performance tasks is required of the students. In addition to the formal nature of math curriculum, the participants agreed that teaching experience offers them an opportunity to deviate from the prescribed sequence in an effort to achieve a more holistic understanding of various math concepts. Bill continued by stating “once you have been teaching for a while I think you can use that experience to change something and not be afraid you are not going to get through the curriculum”. This sentiment was not shared by all participants because of obstacles presented by their building and/or district leadership.

Some participants noted that the vertical alignment of math courses in the secondary school arena has been beneficial for their students. Most of the participants commented on the impact that the school environment and culture can have on the delivery of curriculum. In some cases, Sean noted the resource material contained “ridiculous models…that are setting our kids up for failure”. Rigidity and complexity of the textbooks are creating a heightened focus on procedural fluency while ignoring the need to learn the concepts. Others deviated from the resource material and referenced the curriculum leaders as the greater obstacle to helping students achieve mathematical proficiency. Lisa commented that “you are forced to achieve state standards so our curriculum is designed to achieve higher test scores then we are to foster that much content knowledge”. She continued to reflect on the lack of emphasis on giving students the opportunity to be creative, and the negative impact this can have on developing conceptual understanding. Ultimately, the focus group participants agreed that increased teacher autonomy could lead to greater levels of mathematical proficiency despite the contents of the written curriculum.

The participants agreed that teachers who are given the flexibility to teach beyond the curriculum at times must recognize when and what material to sacrifice. In order to deliver a balanced approach to learning mathematics, Lisa remarked “you want to make your lesson fun and outside of the box; and if you do that and it takes longer to achieve you are sacrificing another topic that might be on a test or state exam”. Other members of the focus group presented the concept of cross-curricular activities that allowed students to experience connections between math and other subjects. Ray remarked on his experience of “go[ing] to the physics or chemistry teacher to see what they were doing and [he] could pull into [his] lessons. Lisa reinforced this notion by commenting “in the rare case that you do get them to line up and they are taking Algebra II and Chemistry at the same time. I have one class of sophomores doing this and it is spectacular and a lot of fun”. The participants discussed these examples as exceptions to the typical operations within their schools focused around curriculum.

The following section is a synthesis of the survey data and the focus group participants' experiences, comments and philosophy as a means to address the research questions that have propelled this study on humanizing mathematics.

Results and Interpretations

The intent of this study was to uncover the root of the discontinuity that exists between the current mandate for a balanced approach to mathematics education and the actual instructional practices of secondary mathematics teachers. The research study conducted was a two-phase, sequential explanatory mixed methods investigation of teachers' perspective on procedural fluency and conceptual understanding with respect to their pedagogical value in secondary mathematics. The study consisted of a first phase of administering a survey instrument to secondary mathematics teachers to collect data pertaining to their valuation of procedural fluency, conceptual understanding, and learning synergy. Following this first quantitative phase of the study, participants engaged in a focus group interview for the purpose of discussing the influence of teachers' valuation on instructional strategies and the factors that lead to the development of identified beliefs found in the survey. The following is a presentation of the results and interpretation of the findings detailed in the previous section.

Several research studies have concluded that teachers' beliefs impact their approach to teaching (Hoyles, 1992; Boaler, 2000; Ambrose, 2004; Kajander, 2007; Kajander, 2010). In the present study, teachers voiced their opinions on two specific strands of mathematical proficiency that have a direct impact on pedagogical decisions surrounding instruction, activity design and the creation of assessments. Table 4-7 is a summary of the data collected from the survey instrument during the first phase of the study. The table suggests the participants were in general agreement with the statements that concerned the aspects of conceptual understanding. The range of mean responses less than one with a mean rating of nearly five is evidence of this agreement.

Table 4-7: Descriptive Statistics for Overall Responses by Construct

Construct	Minimum Mean Rating	Maximum Mean Rating	Range of Mean Rating	Overall Mean Rating	Standard Deviation of Mean Rating
Conceptual Understanding	4.02	4.87	0.85	4.85	0.32
Procedural Fluency	1.63	3.45	1.82	2.61	0.62
Learning Synergy	1.90	4.62	2.72	3.70	0.22

Table 4-8 shows the overall percentages of the responses for each of the Likert-type scale ratings for each of the constructs. Nearly 94% of the responses pertaining to conceptual understanding reside in the two rating groups of Somewhat Agree and Agree. There were four statements that exceeded 97% agreement. Furthermore, two of these four statements resulted in 100% agreement. These two statements, numbers 1 and 18, also address the category of instruction. The survey participants agreed with the fact that students should be able to "understand how and why math procedures work", and that "memorizing the steps" in such procedures should be avoided. The significant degree of agreement clearly indicates the participants' valuation of conceptual understanding in teaching mathematics.

The results of phase 1 are clearly contrasted when analyzing the participants' reactions to conceptual understanding versus procedural fluency. The survey data shown in Table 4-8 indicates just a 29% level of agreement with the statements regarding the overall construct of procedural fluency. With reference to Table 4-2, only one statement was found to have a level of agreement greater than 50%. Statement number 5 relates a student's need to be taught mathematical methods step-by-step. Only 49% of the survey participants agreed with this statement. The participants reported more neutral responses in the procedural fluency construct compared to the other two constructs. Statement number 8 had the lowest degree of neutral statements with one of the lowest mean ratings in the construct. Participants were more disagreeable with this statement that suggests teachers' main objective is to get students to solve problems without the conceptual basis for how or why the solution works.

When considering the entirety of the data collected from both phases of the study, the general disagreement with the statements of procedural fluency is reflected in the focus group's testimony that the teaching and learning of mathematics is grounded in students' aptitude for performing the processes necessary to solve problems. One of the members of the focus group summarized the groups' perspective on the significance of procedural fluency as a pedagogical approach to math instruction by stating "[teachers] push them through with the procedural knowledge". This sentiment is reinforced by the results of a 2004 study by Liu & Thompson. The researchers found that teachers failed to demonstrate acceptance of multiple approaches to solving problems. Through subsequent discussions with teacher participants in the problem-solving workshop, Liu & Thompson concluded that the teachers did not possess the conceptual understanding necessary to lead students beyond the limits of procedural learning (2004).

Table 4-8: Percentage of Overall Responses by Construct

Construct	**% Disagree (1)**	**% Somewhat Disagree (2)**	**% Neither Agree nor Disagree (3)**	**% Somewhat Agree (4)**	**% Agree (5)**	**% Combined Somewhat Agree & Agree**
Conceptual Understanding	0.21	1.07	5.13	28.48	65.10	93.58
Procedural Fluency	23.82	27.90	19.10	21.39	7.78	29.17
Learning Synergy	7.09	13.53	15.46	29.55	34.37	63.92

Participants in the focus group interview admitted that more traditional approaches to instruction offer greater comfort in meeting the demands of a standardized curriculum. One of the interviewees noted that teachers "are forced to achieve state standards so [the] curriculum is designed to achieve higher test scores then we are set to foster that much content knowledge". In a 2013 study investigating the instructional practice of problem-posing, Cai et al. found that the quality of the problems posed by students was congruent with their problem-solving ability. Creativity is not a typical learning objective in mathematics, but the strategy of problem-posing suggests a link between conceptual understanding to student creativity (Cai et al., 2013). Pedagogical practices that encourage more creative thinking are aligned to a more conceptual approach as opposed to the rote practice of procedural activities. In a 2012 study investigation the direct intervention utilizing a new conceptual teaching strategy, Alon found a positive correlation between conceptual teaching and student learning when compared to traditional methods of instruction. Unfortunately, the study does not differentiate the impact on student achievement when comparing the use of the experimental conceptual and the enhancement of the teachers' instructional ability through professional development (Liu & Thompson, 2004; Kang, 2007; Francisco, 2013). The vague distinction may indicate that teacher training could be a factor in the investigation of improved conceptual understanding while minimizing the focus on procedural teaching.

In a similar study to the current research teachers were asked to consider two aspects of being a math teacher. One teacher participant summarized a point of delineation by stating "there is a difference between teaching math and just math knowledge … you have to establish what the big ideas are for students" (In Kajander, 2010, 92). It is supposed that very few prerequisites are necessary to be considered a recipient of the concepts and theories related to mathematics as subject matter. Conversely, the art and science of presenting students with the opportunity to learn math with proficiency is enhanced with a deeper conceptual understanding of the content. The present study has established parallel results to Kajander's 2010 study. Statement 22 of the survey instrument refers to the difference between correctness and a demonstration of understanding the process utilized to determine a solution. The statement reads: "it is more important to me that students answer questions correctly, than it is to demonstrate the process utilized". Only ten percent of the survey participants agreed with this statement. Teachers in a traditional model of instructional practice rely on both curricula and resources, aligned to Common Core Math Standards, to provide students with learning experiences that hope to achieve adequate student performance. A participant in phase two of the current study commented that the resource material being used contains "ridiculous models … that are setting our kids up for failure". When examining the difference between discovery learning strategies in daily lessons and expository learning strategies, a greater level of proficiency among the students was discovered (Mandrin & Preckel, nd). The researchers noted the positive correlation between the increased level of difficulty and the level of improvement was a result of the more challenging material. It should be noted that the material was only considered more challenging because of the mode of instruction; the acquisition of learning was student driven rather than teacher driven. The study conducted by Mandrin and Preckel was unique in its cross-curricular learning environment. In addition to blending curricular areas to enhance educational opportunities, a blend of procedural fluency and conceptual understanding is consistent with current best practices and can lead to a greater level of mathematic development. As students become situated in a setting that is more student-centered and collaborative with the teacher, the classroom leader is expected to possess a greater level of understanding of the material being taught rather than just the ability to demonstrate procedures.

In a 2008 study by Moldavan, teachers' lack of self-efficacy and heuristic understanding in the area of mathematics based on their teacher preparation program were identified as major factors in the NRC's investigation into high quality instructional practices. As an emphasis on new and varied instructional practices encourage cross-curricular learning, teachers of mathematics are recognizing the need for learning synergy with respect to mathematical proficiency as demonstrated in STEM (science, technology, engineering and mathematics) activities. The reference to STEM serves to highlight the innovation skills of the 21st century such as critical thinking and creativity that are becoming more pervasive in K-12 education. Careful and consistent instructional strategies, practice situations and assessment methods all need to be considered when investigating the relationship between STEM and conceptual learning (Kapabinar, 2005; NMAP, 2008; Ernst & Clark, 2012), while also supporting procedural fluency with such tenets of creativity as comprehension, flexibility, and critical judgement (Star, 2005). Although these skills are not explicitly referenced in the survey, some of the participants in the focus group cited creativity as a necessary, yet missing, component of the math learning experience for their students. One participant commented:

> *The bigger piece in my head is they just don't have a lot of creativity in their head. They treat math as a science so they lost the art aspect of mathematics. They don't know how to think outside the box to solve problems. Unless problem 2 is like problem 1 and they build upon that. I see complaints of the proficiency chart in our curriculum and it looks good on paper, but as a professional I know that it is missing something still. There needs to be another area about creativity. Where a student can fully understand a concept.*

The above demonstrates the need for a more explicit curricular structure that encourages, or at a minimum allows, teachers to incorporate activities in their lessons that blend both procedural practice with conceptual reasoning.

Although the survey data, as shown in Table 4-8, indicates only an approximate rate of 64% agreement with statements pertaining to learning synergy, or a blend of procedural fluency and conceptual understanding, there were two statements that garnered 92% agreement (ref. Table 4-3). The high degree of agreement with statements number four and number nine suggests teachers encourage their students to exercise a variety of methodologies to demonstrate their learning of mathematics through the application of reasoning and logic in authentic situations. In a testimonial of support, one focus group participant stated "while you are doing the procedure you are also doing something to understand the concept. I don't think they can go separately and I don't think one goes ahead of the other". Even though the research is inconclusive regarding the sequence of learning from either algorithmic-to-heuristic or its reciprocal (Byrnes & Wasik, 1991; Bednar & Sweeder, 2010), the survey data suggests the participants mostly favor learning synergy. In greater support, the focus group members repeatedly commented on the necessity for the blend of procedural fluency and conceptual understanding, and the sequence of the two cannot be scripted nor necessarily planned.

The data collected in both phases of the current study shows a relative consistency of teachers' valuation of the individual constructs of procedural fluency and conceptual understanding. The construct of learning synergy presented the greatest range in mean rating values on the survey, and also resulted in some vacillation in the opinions of the focus group members. The discrepant results prove to be consistent with prior research, and the culprits are most notably the emphasis on standardization in educational policy and a lack of clarity regarding the sequence of procedural fluency and conceptual understanding. As educators attempt to create a distinct connection between procedural learning and conceptual learning, the standards of mathematical practice presented by NCTM and the Common Core Math Standards must continue to be a major factor in the planning of instruction, activities and assessments (Allen). Canobi (2008) found that conceptual relations help children to extend their procedural fluency beyond the particular problems they have already solved to new problems. Also, children can develop their reportable conceptual knowledge as a result of procedural experience. This change from procedural learning to procedural experience suggests the importance of constructivism, constructionism and situated learning (Boaler, 2000). The challenge for the teachers in the focus group seems to reside in the curricular demands that compact content for the sake of explicit representation of the standards leaving little room for avenues of creativity and critical thinking that move students from the algorithmic to the heuristic condition of learning.

The general consensus of the focus group participants found that recognition of individual students' needs and interests was a necessary first step in planning appropriate and differentiated activities that could address both procedural fluency and conceptual understanding. Although the overall level of agreement with the statements of learning synergy on the survey were not as strong as those for conceptual understanding, the focus group shifted the weight of support in favor of the blended construct. This group, though, did not demonstrate clarity in the posturing between the two strands of mathematical proficiency. One participant noted "I think you need to take a leap of faith, follow the rules and see that it works and once it works and you continue to master that and then conceptual understanding comes". The contrasting viewpoint came from a second participant who offered:

> *I believe that students need to be able to understand the concept before they can really follow procedures. Students who don't understand what they are doing - they can't do anything with it. They can't do any applications with it or get to the next level of math or do anything different with it. I believe they need to have a good understanding of the concepts first.*

Rather than debate the proper – or even the necessity of - sequence, a 2008 NMAP report encourages teachers to "emphasize these interrelations; taken together, conceptual understanding of mathematical operations, fluent execution of procedures, and fast access to number combinations jointly support effective and efficient problem solving". The participants in the current study are not contrasting the need for both procedural fluency and conceptual understanding, but rather there is a recognition of the lack of the latter in favor of the former.

The results of this mixed methods research study provide support for previous research on the topic regarding the coexistence of procedural fluency and conceptual understanding regardless of how they are situated in or throughout a lesson. Gary et al. found that high achieving students focus on flexible procedural understanding of a concept in a 1999 study. Taken a step further, these students were able to manipulate their conceptual understanding in a predictable real world event. Authentic learning can only be achieved if the students are given the opportunity to further develop their acquisition of an algorithm to through personal and meaningful application.

Participants in the survey demonstrated a holistic agreement with the statements of conceptual understanding. The results from the survey pertaining to procedural fluency varied more greatly, and also had lower rates of general agreement. Finally, the survey results regarding learning synergy were the least variable, but did not reflect as clear a degree of agreement when compared to those statements relative to conceptual understanding. Comments from the focus group support the ambiguous agreement with learning synergy. One participant commented "they kind of have to see the path before they can understand what makes it work". Another added "when we are talking about Algebra 1 they are learning the little pieces one by one, and it's not until the end until they put them together to make sense of it". There appears to be a certain amount of experience with practicing with algorithms that teachers believe is necessary. Each student brings their own experience and knowledge into the classroom, which teachers can incorporate into their lesson plans. Building on these experiences and existing knowledge to extend student understanding is largely the constructivist theory of learning. The authenticity of the learning is contingent on the approach taken, and the results of the study indicate that a more focused effort to include the conceptual piece is necessary.

The social component of the learning environment becomes an integral part of the discussion around mathematical proficiency and the humanization of mathematics. The essence of Jean Lave's situated cognition was evident in the focus group discussion. Student learning of both procedural fluency and conceptual understanding is contingent on the situations in which teachers place students to learn. A focus group participant noted that "some kids might not understand 'that way' but you touch upon their experiences and ultimately it comes down to differentiating your instruction". Another admission from the focus group added, "I do agree sometimes you have to group kids who can feed off each other that have that reasoning ability and critical thinking". Taking time to recognize students' needs and interests may be the missing link that should be considered when trying to decipher the proper blend of procedural fluency and conceptual understanding. Given this more holistic profile of the students, teachers can design their instruction to incorporate learning groups that exercise the conditions of situated cognition. Johnson and Galluzzo found that students relate algorithms to the learning group. The learning group can collaborate on a concept with the intention of constructing a shared meaning of the holistic learning experience given the opportunity to express their creativity in a meaningful and tangible demonstration. In short, this concept represents the amalgamation of situated cognition and constructionism.

Summary

In a 2008 report by Tall, three phases of mathematical understanding were found to be critical to a student's mathematics experience. These phases, conceptual-embodied, proceptual-symbolic and axiomatic-formal, intend to fulfill the need for an authentic understanding of a concept resulting from of deliberate practice of learned processes and manipulation of mathematical symbols. The unification of these three phases by Tall coupled with the social learning experience reported by Johnson and Galluzzo support the commentary surrounding learning synergy as a recipe for teachers of mathematics in the secondary classroom. The successful amalgamation of these concepts is suggested as a response to the NRC's desire to shift the educational practice of teaching isolated skills and procedures allowing for greater emphasis on problem solving and sense-making (White-Fredette, 2009). The foundation of the present study has been inspired by the goals of the NRC regarding a progressive paradigm shift of the nature of mathematics as a content area of collaboration, construction and creativity. The findings reported in this chapter suggest the debate over procedural fluency and conceptual understanding is still underway. The survey results were organized into three constructs and three categories. With the exception of the construct of conceptual understanding and the category of instruction, the survey participants offered varied opinions on the value of the constructs of procedural fluency and learning synergy within the categories of activity and assessment. The data analyzed from phase two, the focus group, proffered greater depth to the conversation. The participants were able to verbalize their own experiences and examples that resulted in their perspective on the valuation of the mathematical proficiency strands and the relationship with the pedagogical aspects of the study.

In an attempt to relate the findings of the present study to the findings of previous research studies, the author has uncovered a focused presentation of a need for teachers to invest time in learning their students' needs and interests to help them design their instruction, activities, and assessments. Previous investigations have been synthesized around themes that are consistent with the objectives of the researcher pertaining to the relationship between teachers' values and beliefs regarding effective mathematics instruction and the various learning constructs and instructional strategies. The circumstances for which students are expected to learn mathematics is dependent upon the value with which teachers place on procedural fluency and conceptual understanding in the context of the individual student criteria. This chapter presented the results of the research study that clearly indicated a strong valuation of conceptual understanding, but reported that the integration of both constructs in instruction, activity and assessment is admittedly challenging. The following chapter will conclude the study by answering the research questions that directed the investigation.

Chapter 5: Conclusions and Recommendations

Introduction

The profession of teaching requires an individual to possess and hone a variety of skills in order to accomplish the palpable task of moving students along the continuum of their education. On a daily basis teachers plan instruction, lead students in engaging activities, and analyze student progress through the administration of formative and summative assessments. Each of these tasks are designed to present learning opportunities to the students. In the mathematics classroom, each teachers' approach to meeting the demands of the profession is shaped by their own values and beliefs regarding the various strands of mathematical proficiency. In this study, teachers' valuation of two of these strands - procedural fluency and conceptual understanding – has been examined. Previous researchers have concentrated most of the effort regarding these strands toward elementary teachers. The research suggests that elementary teachers are more inclined to emphasize procedural learning. Furthermore, researchers contend that elementary teachers demonstrate a particular avoidance of the more profound strand of conceptual understanding due to the abstract nature of mathematics. When exploring the research with regard to secondary mathematics teachers, the studies were often designed to investigate a particular set of strategies related to mathematical concepts and student performance, rather than any relationship among the strategies and the strands of mathematical proficiency.

The purpose of this study was to examine the value that secondary teachers place upon two learning constructs within the framework of mathematical proficiency, and how this valuation influences their approach to teaching. The study required two phases. In phase one, 58 participants completed an on-line survey instrument consisting of 27 items in order to demonstrate their beliefs regarding conceptual understanding, procedural fluency, and a blend of the two constructs. The second phase of the study invited a small group of survey participants to engage in a focus group interview. The focus group included six participants, and commenced with the intent to offer the participants the opportunity to discuss the rationale, built upon experiences and examples, for the manner in which they value conceptual understanding, procedural fluency, and the blended construct. The context of the survey and the focus group interview resided within the auspices of the Common Core Math Standards in the specific geographic region of southeastern Pennsylvania.

Two learning theories governed the direction of this study on humanizing mathematics. These learning theories, situated cognition and constructionism, offered the researcher a degree of suppleness in the structure of the data collection protocol to not rely on specific pedagogical strategies, but rather authentic contributions from the participants. Situated cognition is a theory developed by Jean Lave that contends learning is dependent on the activity and social context and culture in which it occurs (Lave & Wegner, 1991). The research conducted by Lave and others suggests that learning is a social experience that demands the sharing of knowledge and collaborative problem solving. This model of education is in stark contrast to the notion of learning concepts in isolation without heuristic pursuits.

The second theory of learning is Seymour Papert's constructionism. Constructionism is a social learning theory that developed from Jean Piaget's philosophy of education known as constructivism. The difference between constructionism and constructivism is realized in the manner in which knowledge is actively and tangibly constructed by the student. Papert and Harel (1991) summarize constructionism as the "context where the learner is consciously engaged in constructing a public entity, whether it's a sand castle on the beach or a theory of the universe". Although previous research captured the plausibility of new instructional strategies in the vain of constructionism as a means to student achievement, the present study is focused on the development and utilization of such strategies with regard to the two specific constructs of conceptual understanding and procedural fluency.

The need for such development in the mathematics classroom has been discussed for nearly two decades now, yet math education is still fixated on procedural learning rather than conceptual understanding and procedural fluency (Kilpatrick et al., 2001; NRC, 2005; OECD, 2013; Schoenfeld, 2013; Anderson et al., 2015; TIMMS, 2015). Kilpatrick et al. present conceptual understanding as the "comprehension of mathematical concepts, operations, and relations" (2001, p. 5). Conceptual understanding, also characterized as heuristic learning, is identified by high level thinking intended to form abstract representations of the structures that guide mathematical discovery, while also establishing relationships among those structures. Procedural fluency is the skill of "carrying out procedures flexibly, accurately, efficiently, and appropriately" (Kilpatrick et al., 2001). Procedural fluency, also considered algorithmic learning, has been traditionally regarded as the principal approach to K-12 mathematics education. The focus of learning is grounded in skill development with a secondary concern for increasing efficiency.

The methodology for this investigation was organized around three research questions.

Central Question: How do secondary mathematics teachers value procedural fluency and conceptual understanding in their instructional practices?

Sub-question 1: How do secondary mathematics teachers' valuation of procedural fluency and conceptual understanding influence their pedagogical decisions?

Sub-question 2: How do teachers develop their value-system of procedural fluency and conceptual understandings in their instructional practices?

The researcher utilized a survey instrument to garner 58 participants' opinions on statements related to the two learning constructs to aid in answering the central question. In a secondary phase of the study, six participants volunteered to engage in a conversation designed to explicate the results of the survey. This discourse provided the evidence necessary to address the two sub-questions of the study.

The data collected and subsequent analysis indicated a shift in the focus of math education from developing procedural skills to a necessary emphasis on conceptual understanding. The survey responses provided clear support for the importance of teaching mathematical concepts in conjunction with, and based on some statements, as a substitute for procedural fluency. Although not as dramatically disagreeable as they were agreeable with statements of conceptual understanding, the survey respondents did not favor procedural fluency as nearly critical to math instruction. Finally, the blended construct of conceptual understanding and procedural fluency referred to as learning synergy disclosed a somewhat balanced opinion from the survey participants. Despite most statements finding respondents more agreeable than disagreeable, the overall mean rating was mostly influenced by responses that were neither agreeable nor disagreeable.

The findings from phase two helped inform the survey results regarding learning synergy. Throughout the focus group, participants were found to shift their viewpoints on the blending of the two strands of mathematical proficiency as they presented examples and experiences to rationalize their perspectives. The primary result of the focus group was that both constructs are important, and the sequence and frequency with which teachers incorporate procedural fluency and conceptual understanding is predicated on the mindset of the students and circumstances of their learning. The findings of this study provide answers to the research questions in the subsequent section.

Conclusions

The central research question for the present study examined teachers' valuation of procedural fluency and conceptual understanding in their instructional practices. The data collected by the participants in the survey was analyzed by examining the measures of central tendency. The statements in the survey were organized according to three constructs: procedural fluency, conceptual understanding and a blend of the two. Phase one of the study yielded a clear distinction between the valuation of procedural fluency and conceptual understanding. The survey participants indicated strong agreement with statements related to the necessity to teach mathematics conceptually. The survey data, as represented in Table 4-8, suggests that the participants believe conceptual understanding is a more valued construct when compared to either procedural fluency or learning synergy. A 94% agreement with the statements relating ideas about conceptual understanding versus a 29% agreement with the statements relating ideas about procedural fluency leads the researcher to conclude that the participants in this study favor the former in their instructional planning, activity design and assessment creation.

A comparison of just the two strands of mathematical proficiency eliminate the possible event that a blend of both constructs exists. The researcher embedded statements in the survey to elicit a response from the participants regarding a learning synergy of conceptual understanding and procedural fluency. The overall results of the survey do not suggest overwhelming support for this third construct. Several statements did prove to be highly favored by most participants, but the overall valuation of learning synergy did not find the level of agreement to exceed 64%.

This data point concurs with 2001 Kilpatrick publication noting simply that the strands are not separate.

The unbalanced valuation of the three constructs sits uneasily among some of the participants. A small group of teachers and curriculum administrators noted their consternation regarding the discrepant results throughout their engagement in the focus group interview. Their commentary provided responses to the two sub-questions that aided in the direction of this study.

Each of the two sub-questions were asked in search of an explanation of the data reported by the survey participants. The survey results offer insight into the particular statements utilized to uncover participants' perspectives on the strands of mathematical proficiency. A representative group of respondents were invited to participate in the focus group interview to explore the survey results on a deeper level. The themes of the interview session unveil a collection of how opinions toward the various constructs influence their pedagogical decisions, and develop their value-system toward the various constructs. One participant in the focus group surmised "I don't know that people who get math easily make the best teachers … they never struggled, and therefore, they can't understand what it is like when a kid struggles and they don't understand the mistakes that you make". National reports over the last ten years have concluded that there is a lack of equilibrium between procedural fluency and conceptual understanding in school mathematics (Kilpatrick, 2001; NMAP, 2008). The results of the survey in the present study suggest a false dichotomy exits between the two constructs, and the third construct of learning synergy is not necessarily the variable that balances the equation between the two strands of mathematical proficiency. In the 2001 Kilpatrick et al. report, the authors note that the scale can be leveled through a requisite degree of skill development when attempting to learn most mathematical concepts. In symbiotic fashion, the practice of mathematical procedures can, in turn, help develop conceptual understanding.

Furthermore, Martin notes that students who do not have the conceptual background for the utilization of the procedures, may either apply the procedure inappropriately or use an incorrect procedure altogether (2009). This disconnect can lead to a breakdown in the foundational skills required for the successful attainment of Algebra in either middle school or high school. A return to the tools for practicing mathematics developed by George Polya and Henri Poincaré creates a bridge between the algorithmic and heuristic approaches to learning mathematics. This bridge is an impossibility when we consider the two approaches in isolation, but when taken together a strong foundation can be constructed. Tseng et al. (2011) and Liu and Thompson (2004) discovered that a strict emphasis on algorithmic teaching and learning neglected aspects of social constructionism that included empathy, collaboration and incrementation in the learning process.

The tenets of Lave's social cognition and Papert's constructionism consolidate the strands of mathematical proficiency into an efficient model of mathematics instruction that humanizes each episode of the learning process. The concepts of hands-on learning, problem-solving and creative-thinking are all components of the relationship between conceptual understanding and procedural fluency. In the absence of this constructionist approach, students learn algorithms as an abstract set of procedures that can be applied to a finite set of circumstances (Boaler, 2000). The data from the current study can be summarized with regards to humanizing mathematics in the following statement from one of the focus group participants:

> *[The students] don't care about math or Algebra; they just want to be kids. We are doing our best to try to crack that code and figure out that maze of what is going on inside that*

head. So, when we are sitting here as professionals and are trying to come up with the best plans to figure out the strategies … it's about that connection with the kid seeing what they know.

A strong foundation in mathematics skills cannot be secured by a single strand of mathematical proficiency, but rather by the interweaving of multiple strands. The layering of procedural fluency with both the salutation and rationale provided by conceptual understanding has been revealed as the secret ingredient to a palatable mathematics education.

Recommendations

The seasoning of mathematics learning can be refined utilizing a variety of methods. The sequence in which teachers introduce, emphasize and review procedural fluency and conceptual understanding can shape a mathematics lesson in its efficient attempts at the transferal of knowledge to the students. Teaching and learning is a dynamic, symbiotic and episodic sequence of interactions predicated on a human relationship between teacher and student. The stagnant collection of data via a survey provided the researcher significant information regarding the valuation that teachers place on the learning constructs of procedural fluency and conceptual understanding. The opinions of the participants were offered in the absence of their relational counterparts – the students. There are five recommendations that the researcher can offer as a result of this study. The first recommendation is an additional method of investigation through observation to gain another level of data regarding teachers' valuation of conceptual understanding and procedural fluency, and the how the valuation influences pedagogical decisions. The second recommendation indicates a need to clarify the purpose of mathematics education as it relates to the development of students as being mathematicians or being mathematically literate. The remaining three recommendations consider the totality of the strands of mathematical proficiency. The third recommendation suggests further investigation into the notion of storytelling, as noted during the focus group interview, as an instructional method, and the potential correlation to the strand of productive disposition. The fourth recommendation suggests that a similar, yet more in depth, study be conducted to measure the presence of each of the five strands in mathematics classrooms. The fifth and final recommendation proposes that a sixth strand, contextual relationship, be considered as a new component of mathematical proficiency as a necessary aspect of teachers' pedagogy when attempting to humanize mathematics.

Recommendation 1: Classroom Observations

The present study was executed utilizing two methods of data collection: a survey instrument and a focus group interview. Participants were asked to share opinions of agreement or disagreement relative to statements pertaining to conceptual understanding, procedural fluency and a blend of the two considered learning synergy throughout this study. The data indicates a strong agreement with the conceptual understanding strand, and a contrasting yet similarly strong opinion of the procedural fluency strand. Although a small sample of the survey participants were invited to engage in dialogue to explore the survey data results, the education, experience and motivation for the survey responses remains widely unknown. In order to explain the survey results more fully, each teachers' history, circumstances and instructional practices should be examined. The National Research Center (NRC) confounds the issue of teachers' non-pedagogical influence on student learning by also identifying curriculum content, learning processes and teachers' education as contributors to students' challenges to learning (Byrnes & Wasik, 1991; Moldavan, 2008). A more explicit means to gathering data for each of the survey participants would be through direct observation.

Observations of instructional practices of teachers provide discerning data regarding evidence of mathematical proficiency (Good & Dweck, 2006). The NCTM (National Council of Teachers of Mathematics) provides a framework for structuring an effective observation. Lessons in a secondary mathematics classroom are often complex, and therefore, observations of such episodes can be overwhelming. Utilizing the framework, known as the Common Core State Standards of Mathematical Practice, or more simply the Mathematical Practices, can help narrow the focus of the observer to hone in on evidence of conceptual understanding and procedural fluency. The following are the eight Mathematical Practices:

1) Make sense of problems and persevere in solving them
2) Reason abstractly and quantitatively
3) Construct viable arguments and critique the reasoning of others
4) Model with mathematics
5) Use appropriate tools strategically
6) Attend to precision
7) Look for and make use of structure
8) Look for and express regularity in repeated reasoning

In order to further simplify the process of observation with an emphasis on witnessing a correlation between survey responses and teacher practice, the researcher would rely on the following three interdependent dimensions of mathematics classrooms: 1) knowledge of mathematics content, 2) enactment of learning and pedagogy, and 3) facilitating intellectual community (Kanold, Briars & Fennell, 2012).

The focus group discussion revealed some insights into the rationale for the agreement with statements of conceptual understanding and disagreement regarding statements of procedural fluency. By pursuing the following inquiry, the survey participants' valuation could be verified through direct observation to further address the research questions of this study.

- Who is doing the mathematical thinking – teachers, students, or both?
- What is the goal of the instruction – understanding mathematics or simply getting answers?
- What is the cognitive demand of the tasks students are being asked to do?
- What happens to the demand as teachers introduce the task, and as students work on the task?
- To what extent are all students engaged in mathematics learning?

The answers to these questions through observation could then be correlated to the responses of the survey, and the comments from the focus group. The survey participants' valuation of conceptual understanding and procedural fluency could be validated or refuted through instructional practices, activity design, and assessment administration.

Recommendation 2: Mathematicians or Mathematical Literacy

Throughout this investigation on humanizing mathematics, previous research studies and current participants' perspectives both implicitly and explicitly referenced standards for mathematical practice. This second recommendation indicates a need to clarify the purpose of mathematics education as it relates to the development of students as being mathematicians or being mathematical literate. The Common Core State Standards Initiative clearly states that both conceptual understanding and procedural fluency are important and deserve equal attention in the mathematics classroom. It is recommended to study the motivation of the advocates of the Common Core, authors of textbooks, developers of curriculum, and designers of standardized assessments with respect to mathematical literacy.

Larson defines mathematical literacy as “student development of skills and procedures, conceptual understanding, problem solving, and a disposition to expend effort and persevere when learning mathematics and solving problems” (2016). This definition very nearly summarizes the five strands of mathematical proficiency. On the other hand, according to the United States Department of Labor mathematicians “conduct research to develop and understand mathematical principles”, and also “analyze data and apply mathematical techniques to help solve real-world problems” (2015). There is an overlap in the description of mathematical literacy and mathematicians. The PK-12 lens should be applied to how the teaching and learning of mathematics is aligned to developing mathematical literacy or mathematicians. The focus of the decision-makers in the PK-12 learning environment - with respect to the delineation of these two approaches - is suggested to be critical to the nature of mathematics learning. Recognizing the distinction, and the subsequent concentration on developing either mathematical literacy or developing mathematicians, is expected to have an impact on the implementation of the strands of mathematical proficiency that are discussed in the following recommendations.

Recommendation 3: Story Telling

One of the arguments presented during the focus group interview highlighted an aspect of mathematics education that is often missing from curriculum documents and resource materials. A focus group participant commented that “[w]hen our kids graduate they would say things they remember the most about the class were the stories and shockingly they would remember the concepts behind the stories”. It is supposed that the practice of storytelling in mathematics finds little air time because of the depth and complexity of the math content in secondary classrooms. The example presented in the midst of the current study was that of the enigmatic arrangement between Marquis de L’Hospital and John Bernoulli regarding the topic of L’Hospital’s Rule, a concept that garners procedural real estate in most calculus textbooks negating the fact that the rule was discovered by Bernoulli, and subsequently purchased and published by L'Hospital.

The researcher has experienced success in this practice of storytelling when teaching lessons on complex numbers, and in particular, the imaginary number *i*. Although the story of imaginary numbers has a long history, mathematicians Girolamo Cardano and Raphael Bombelli were at the forefront of its use and subsequent dislike of the concept. Cardano and Bombelli were both Italian polymaths studying mathematics in the 1500s. The discovery of imaginary numbers was not well received by the larger audience because it was difficult to describe and explain on a conceptual level. Through the centuries, notable mathematicians like René Descartes, Leonhard Euler and Carl Friedrich Gauss continued to utilize the number *i*, and worked to find ways to encourage its use and understanding among others.

The construction of the idea of complex numbers was the ultimate discovery that started to bring understanding of imaginary numbers into view. The story of mathematical concepts adds realism to lessons. The process of incorporating imaginary numbers into a complex solution can be readily accomplished. Knowing the centuries long collaboration that mathematicians endured could provide a cognitive and emotional connection between students and their learning. A follow-up study to measure the correlation between conceptual understanding and storytelling in mathematics could provide information for an additional instructional strategy that should be incorporated throughout curricula, resource materials and lesson plans.

Recommendation 4: Investigate All Five Strands

In 2001, the National Research Council released the report that introduced the five strands of mathematical proficiency. The current study examined teachers' perspectives on two of those strands: conceptual understanding and procedural fluency. These two strands contain the primary utilities of mathematics education, but the NRC persisted in the establishment of proficiency through strategic competence, adaptive reasoning and productive disposition to complete the quintet. This study can be enhanced through the examination of the latter three strands in similar fashion. The survey statements were designed to measure teachers' valuation of the two constructs individually and collectively. The survey could be expanded to include strategic competence, adaptive reasoning and productive disposition in order to determine how the five strands are interdependent and interwoven, and how they influence teachers' decisions regarding instructional delivery, activity design and assessment administration.

According to the NRC, "strategic competence refers to the ability to formulate mathematical problems, represent them, and solve them" (2001). When observing this strand, one should expect to see students engaged in both problem finding, problem posing and problem solving. Strategic competence demands that teachers challenge students to identify a problem based on realistic parameters. Once a problem can be formulated, a mathematical representation of some kind is the next step in this strand. Students need to rely on mathematical symbols, manipulatives, and constructions to transfer the problem situation into a mathematical expression that can be solved through the application of procedural skills. Once the form of the problem fits the structure of an algorithmic condition, a student's conceptual understanding can be applied to proceed to solving the problem. Strategic competence, with procedural fluency and conceptual understanding, provides students with a tool to solve non-routine problems after practicing the processes found in routine problems and exercising understanding the fundamental concepts of algebra. Computing a solution is not a sufficient conclusion to a problem, and therefore, the NRC included adaptive reasoning as the fourth strand.

Teachers' valuation of adaptive reasoning in the secondary mathematics classroom could aid in the research to quantify the building of students' "capacity to think logically about the relationships among concepts and situations" (NRC, 2001). Adaptive reasoning demands that students first consider the various algorithmic approaches to solving a problem, apply procedural skill, and construct a viable argument to justify the resulting solution. This strand raises the level of complexity of mathematics by extending the requirements of students beyond just procedures and concepts. Adaptive reasoning is a concluding component of problem solving. The NRC's report weaves together adaptive reasoning with the other strands of mathematical proficiency by noting "[l]earners draw on their strategic competence to formulate and represent a problem, using heuristic approaches that may provide a solution strategy" (2001). Students determine the validity of the chosen strategy through adaptive reasoning. Furthermore, adaptive reasoning with conceptual understanding provides students with the tools for justifying solutions. The requirement of reasoning complicates the process of mathematics teaching and learning, but the fifth strand provides a shock of stability through sense-making.

The final strand in the NRC's report is productive disposition, which "refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (2001). Similar to the challenges the mathematicians faced in the 16^{th}, 17^{th} and 18^{th} centuries regarding imaginary and complex numbers, students are more likely to struggle in learning mathematics unless they believe it to be understandable and applicable. The survey participants indicated a high level of agreement with the suggestion that teachers encourage their students to exercise a variety of methodologies to demonstrate their learning of mathematics through the application of reasoning and logic in authentic situations. This notion resembles the ideals of adaptive reasoning and productive disposition.

An aspect of teaching and learning that can be explored through an investigation of perseverance is productive disposition. Teachers' valuation of this strand could allow for the researcher to discover a correlation between the accumulation of mathematical concepts and students' ability to learn more complex procedures and to develop confidence in their ability to reason logically. The importance of the NRC's portrait of mathematical proficiency can be summarized in the statement "[s]tudents' disposition toward mathematics is a major factor in determining their educational success" (2001). An extension of the present study to include all five strands of mathematical proficiency could address both how teachers value each strand, but also how often they incorporate each of them in their lessons.

Recommendation 5: The Sixth Strand of Mathematical Proficiency

A 2015 report by Anderson, Valero and Meaney found that 16-year-olds described their attitude toward "their affective relationship to mathematics" as "bored". Instructional strategies involving creativity and problem solving require students to absorb procedural knowledge, process the new knowledge, and then utilize that knowledge as a means to progress in their mathematics education. The five strands of mathematical proficiency address the need for teachers to engage students through the practice of procedural skills, development of understanding, problem finding and problem solving, reasoning and sense-making. One significant aspect of the teaching and learning dichotomy that is not addressed in the current structure of the interwoven strands of mathematical proficiency is that of contextual relationships. The fifth and final suggestion is a matter of studying what teachers think of the need for developing teacher-to-student relationships, how such relationships can be established and evolved, and how these relationships can help to eliminate boredom in the math classroom.

A particular strategy for engaging students and developing relationships with students could be studied through the incorporation of STEM activities. Relating to a theme of the current study, students demonstrating greater conceptual understanding have been found to show an affinity toward STEM (Science, Technology, Engineering, and Mathematics) related activities such as: creative social play, artistic design, technology utilization and block construction (Eden & Potter, 2012). These activities are consistent with the variety of theories and activities previously identified; namely, Lev Vygotsky's social constructivist theory and Seymour Papert's constructionist learning activities to enhance conceptual learning in mathematics (Tseng et al., 2011; Edens & Potter, 2012; Francisco, 2013). Likewise, the theory of situated cognition relates directly to the strands of mathematical proficiency, as well as, the need for notable contextual relationships to be formed.

The five strands could be strengthened by a sixth component requiring teachers to extend their lessons beyond those of computation, understanding and logic. The forging of relationships could prove to create an even more authentic learning environment for the student that not only includes sense-making on a personal level, but also the importance of mathematics and its application to the social collective of their community. Contextual relationships in mathematics could be a key ingredient to achieving true mathematical proficiency. One of the focus group participants engaged with her students by "giv[ing] them a chance for feedback … listen[ing] to what they have to say". She continued to note that this simple interaction made them "feel like they are invested", and the students acknowledged the teacher's interaction when commenting "she listened to my idea". There is a partnership in education that resides among the best practices of teachers. There is a humanism in working together to teach and learn and become more mathematically proficient.

Summary

The evidence presented in the previous chapter regarding teachers' perspectives on the value of conceptual understanding and procedural fluency in secondary mathematics appears to support the perpetual pendulum swing that Matt Larson references in his article "The Elusive Search for Balance". Abbreviating Dr. Larson's retelling of the history of mathematics in the United States, the first major U.S. mathematics textbook by Nicolas Pike in 1788 prescribed a sequence of stating a rule, showing an example, and having students practice problems similar to the example. Lessons in mathematics began with the procedure. Warren Colburn was the first to upset this manufactured approach to learning math when he introduced a series of texts starting in 1826 that recommended teachers present students with the materials and the opportunity to discover mathematics. The conceptual method did not last long, when in 1831 a new publication encouraged direct instruction of procedures. The advent of "New Math" arose in the 1950s, emphasizing student development of mathematical concepts and reasoning. Math education returned "back to the basics" beginning in the 1970s, and remained focused on procedural skills and fluency until the evolution of standards-based education was born out of the National Council of Teachers of Mathematics in 1989. The NCTM standards became widespread throughout the United States over the next several years. The pendulum reversed direction one more time, and the nation has embraced the notion of procedural fluency though direct instruction and explicit practice of skills in lieu of conceptual understanding. The survey results of the current study seem to indicate that the pendulum is near the apex, and is preparing to begin its decent to continue the unrelenting oscillation. The commentary offered by the focus group participants represents an obstacle to the anticipated paradigm shift that has been repeated for over 200 years.

This research study has suggested that teachers of mathematics may be poised to surrender their monogamous relationship with either one of the strands of mathematical proficiency, and embrace the concept of learning synergy. Traditional mathematics education has an overwhelming dependence on convergent thinking (Kilpatrick, 2001; NMAP, 2008). Recognizing the swing of the pendulum is a challenging task for those teachers that have been practitioners for several years, and are mired in an emphasis on procedural fluency. The advent of new educational paradigms such as STEM can be intimidating as they threaten to interrupt the forward momentum of the pendulum. As teachers begin to recognize that they may be pulling too hard on one strand, an abrupt change could be damaging. A sudden change from basic algorithmic responses can be daunting for students, and result in a loss of self-confidence. "The desire to achieve understanding in a technical subject such as mathematics while minimizing the component of skills is a most human one" (Wu, 1999). Careful adjustments in the classroom, and a holistic attempt to develop instructional strategies that approach a synergy of conceptual understanding and procedural fluency are likely to promote mathematical proficiency for the students.

Although the survey data in this study suggests that teachers may be viewing the next change in the pendulum's directions to reduce efforts in developing procedural fluency in lieu of conceptual understanding, the more explicit testimony from the focus group is in agreement with the 2014 NCTM report noting that students' progress in conceptual understanding should not consist of isolated experiences from procedural exercises, but rather in conjunction with instruction on procedures. Curriculum developers, textbook authors and teachers alike should strive for equilibrium between the two forces of procedures and understanding. Hung-Hsi Wu concludes his 1999 article by pleading “[l]et us teach our children mathematics the honest way by teaching both skills and understanding”. In order to humanize mathematics, the pendulum that is powered by the false dichotomy of procedural fluency and conceptual understanding should come to rest.

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APPENDIX A: DESCRIPTION OF SCHOOL DISTRICTS

In order to preserve anonymity, the school district names have been replaced with an alphabetic notation. The data for the following table was compiled from the Pennsylvania State school performance website http://paschoolperformance.org and from the various school district websites. All websites were accessed on May 9, 2016.

District	# of Students	# of Math Teachers	# of Secondary Schools
A	5,653	47	5
B	2,650	18	3
C	9,703	93	8
D	3,212	28	3
E	2,381	31	2
F	2,153	20	2
G	1,139	15	2

APPENDIX B: PERCEPTIONS OF MATH (POM) SURVEY

POM Values Questions

Please answer these questions by circling the response, where 0 is low or poor or disagree, and 3 is high or positive or agree. Please do not add other responses such as “not sure” – choose the closest response to your feeling.

1)	It is important to me to be able to get the correct answer to math questions.	0	1	2	3
2)	It is important to me to really understand how and why math procedures work.	0	1	2	3
3)	It is important for everyone to be able to accurately do basic math calculations such as addition or multiplication, without a calculator.	0	1	2	3
4)	Everyone needs to deeply understand how and why math procedures work if they are going to make effective use of them.	0	1	2	3
5)	It is important to be able to recall math facts such as addition facts or times tables quickly and accurately.	0	1	2	3
6)	It is important to have to think through and understand a variety of different approaches to problems.	0	1	2	3
7)	It is the teacher’s job to teach the steps in each new math method to the students before they have to use it.	0	1	2	3
8)	There are often several correct ways to get a right answer.	0	1	2	3
9)	Accurate and efficient calculation skills are highly important in mathematics.	0	1	2	3
10)	It enriches student understanding to have to think about different ways to solve the same problem.	0	1	2	3

11)	It is important to practice on many familiar shorter math questions in school.	0	1	2	3
12)	It is important to develop connections between related ideas and models in mathematics.	0	1	2	3
13)	Most people learn math best if they are taught the methods step by step.	0	1	2	3
14)	When I'm learning math I really want to know "how" and "why" the methods and ideas work.	0	1	2	3
15)	Calculators shouldn't be used too much in school because they can lessen opportunities to practice computational skills.	0	1	2	3
16)	Children learn deeply by investigating new types of problems different from ones they've seen before.	0	1	2	3
17)	There is usually one best way to write the steps in a solution to a math question.	0	1	2	3
18)	Most people learn math best if they explore problems in small groups to discuss and compare different approaches.	0	1	2	3
19)	Learning to follow "the steps" to generate correct answers is very important.	0	1	2	3
20)	It is important to develop connections between ideas by working on multi step problems.	0	1	2	3

APPENDIX C: BELIEFS AND VALUES FOR TEACHING MATHEMATICS SURVEY

Beliefs and Values for Teaching Mathematics Survey

Demographics

Please circle the most appropriate selection.

Gender M F Grade(s) Taught 7 8 9 10 11 12

School or school district: ____

Teaching Certification ☐ Instructional I (Elementary) ☐ Instructional I (Secondary Math)

☐ Instructional II (Elementary) ☐ Instructional II (Secondary Math)

Years of experience teaching algebra content (please indicate the years you taught secondary math. If you are still teaching, the end date should be "present").

I taught mathematics courses with Algebra content from ___________ to ____________.

Please indicate your level of agreement or disagreement with the following statements by circling the most accurate response based on your experience and your values as a teacher of mathematics. The scale used for the responses of 1 – 5 is: 1 (Disagree), 2 (Somewhat Disagree), 3 (Neither Agree nor Disagree), 4 (Somewhat Agree) and 5 (Agree).

		Disagree				*Agree*
1)	It is important to me that my students really understand how and why math procedures work.	1	2	3	4	5
2)	It is the teacher's job to teach the steps in each new math method to the students before they have to use it.	1	2	3	4	5
3)	Most people learn math best if they explore problems in small groups to discuss and compare different approaches.	1	2	3	4	5
4)	I allow students to use diagrams or sketches to demonstrate their learning to me or to other students.	1	2	3	4	5
5)	Most people learn math best if they are taught the methods step-by-step.	1	2	3	4	5

6)	Students who don't remember a formula for a particular problem on a math test will not be able to solve the problem.	1	2	3	4	5
7)	When students ask questions, I try to suggest a line of thinking so they can construct a solution on their own.	1	2	3	4	5
8)	I've met my teaching objectives if students can solve problems on a particular topic, even if they don't remember how and why their solution works.	1	2	3	4	5
9)	My students should be able to apply the reasoning and logic skills learned in math to their everyday lives.	1	2	3	4	5
10)	My students should be able to deduce reasonable answers while solving mathematical problems.	1	2	3	4	5
11)	In order for my students to learn math, they need to memorize specific necessary information.	1	2	3	4	5
12)	It is important to practice on many familiar shorter math questions in school.	1	2	3	4	5
13)	In order to be successful when attempting complex problems, students need to master basic operations first.	1	2	3	4	5
14)	Students who know when to apply a particular formula to solve a problem, do not need to know the underlying concepts that explains why the formula works in that particular situation.	1	2	3	4	5
15)	Most math activities and exercises should include a variety of assessment strategies, some of which are not necessarily predicated on finding the correct solution.	1	2	3	4	5
16)	To be successful, students should have strong procedural skills despite weak conceptual understanding.	1	2	3	4	5
17)	The lessons I teach typically involve multiple mathematical concepts within a single focused objective.	1	2	3	4	5
18)	I encourage students to relate new material to already acquired knowledge or past experiences, rather than just memorizing the steps to a practiced solution.	1	2	3	4	5

19)	In mathematics, students usually only need to learn to solve problems using one method.	1	2	3	4	5
20)	Students usually struggle to apply new procedures to unique problems even though they have already demonstrated an understanding of how to solve related example problems.	1	2	3	4	5
21)	Students should be able to construct formulas and/or procedures to solving problems in order to demonstrate their understanding.	1	2	3	4	5
22)	It is more important to me that students answer questions correctly, than it is to demonstrate the process utilized.	1	2	3	4	5
23)	Students should trust their solutions as long as they utilized the correct method to solving the problem, even if the solution is contrary to the answer they were expecting.	1	2	3	4	5
24)	It enriches student understanding to have to think about different ways to solve the same problems.	1	2	3	4	5
25)	Students should be able to construct explanations of their mathematical ideas without the use of formulas or equations.	1	2	3	4	5
26)	Students should be given a lot of opportunity to practice computational skill, rather than relying on a calculator.	1	2	3	4	5
27)	Students should be able to construct a solution to a math problem using a variety of methods, rather than relying on a single method.	1	2	3	4	5

Following this first part of my research study, I will be assembling a small group of teachers (6-8) to participate in a focus group interview to further discuss the responses that you provided in the survey. Please indicate your interest and willingness to participate in the focus group. If you are interested, you will be contacted by either email or phone to discuss the meeting date, time and location.

☐ Yes, I would like to participate in the focus group. I can be reached at the following:
Email address ____
Phone number ____

☐ No, I am not interested in being part of the focus group.

APPENDIX D: PERMISSION TO USE AND MODIFY POM

Dr. Kajander,

Hello. My husband is currently enrolled in Drexel University's (Philadelphia, Pennsylvania, USA) Doctoral program with a concentration in creativity and innovation. He is a mechanical engineer trained high school math teacher and is working on his dissertation. He is researching instruments that he could use in his study focused on secondary education teachers and their beliefs and attitudes toward conceptual vs. procedural understanding. Your instrument was brought to our attention and we were hoping you could share it with us and let us know about the research on the instrument.

POM (Perceptions of Mathematics) This instrument was designed to examine knowledge as well as beliefs about knowing and teaching mathematics. Specifically, procedural and conceptual knowledge are separated. A Profile graph of individuals' scores is provided by the instrument to encourage participants to self-reflect about the relative positions of their belief in the value of conceptual learning compared with their own knowledge in this domain.

Thank you for your consideration and we have appreciated reading your work. My husband hopes to make a contribution to math education through his work with teachers.

Thank you,

Katie

From: Ann Kajander
Sent: Tuesday, March 8, 2016 9:19 AM
To: Katie Kennedy-Reilly
Subject: Re: Conceptual vs. Procedural Understanding Study

Thanks so much for your inquiry.

By all means the survey would be available. We have used it in house to support our own program development. Personally I have found it helpful.

I wonder if it would help if we had a chat on the phone? (or perhaps your husband might want to). I am currently in the US on sabbatical, so the number 863 425 5774 might be relatively cheap for you to call? Feel free to try anytime but late afternoon might be the most reliable to get me in. Sometimes my husband has the phone ...

There isn't much out there, so it may be a starting point ...

Sending along two papers. I can also send the full survey with scoring guide if interested.

Happy to talk more,

Ann

On Tue, Mar 8, 2016 at 6:56 PM, Brian Reilly <bmr67@drexel.edu> wrote:

Good evening Dr. Kajander,

I wanted to personally thank you for the information that you shared with my wife, Katie, earlier today. She forwarded to me your message with the attached articles. I believe the POM Survey that was included in the "Unpacking Mathematics ..." article targets the desired data for my proposed study. I appreciate the invitation to speak with you, and would like to schedule a day/time that would be convenient for you. You also mention the opportunity to get a hold of the full survey and scoring guide. I would greatly appreciate access to those documents as well. If you are available, would I be able to give you a call either tonight still (I apologize for being forward on short notice) or tomorrow afternoon (eastern standard time) to discuss my objectives and the advances that you have experienced in your own work.

Once again, your assistance and openness are appreciated, and I look forward to speaking with you.

Brian

From: Ann Kajander
Sent: Tuesday, March 8, 2016 8:10 PM
To: Brian Reilly
Subject: Re: Conceptual vs. Procedural Understanding Study

Sure.
How is tomorrow around 3:00 or 3:30?

Ann

Ann Kajander
Associate Professor
Faculty of Education
955 Oliver road
Lakehead University, Thunder Bay, ON
P7B 5E1
(807) 252-1110

Please rescue a dog rather than supporting the puppy mill industry. Check out Petfinder.com

APPENDIX E: FOCUS GROUP PROTOCOL

Focus Group Interview Protocol

- Prior to beginning the interview, thank the participants for attending;
- Remind the group of the topic of the interview and the interview length;
- Acknowledge the visual showing the definitions for the five strands of mathematical proficiency that will be used to clarify the terms "conceptual understanding" and "procedural fluency";
- Inform the participants that the interview will be recorded using an audio and video recording device;
- Inform the participants that pseudonyms will be used, and therefore, they should feel comfortable in engaging an respectful, open, and honest discussion;
- Remind the participants that their participation is completely voluntary, and they may stop the interview at any time;
- Ask the participants to introduce themselves, and provide verbal consent to participating in the focus group interview.
- Lastly, ask the participants to identify themselves (first name only) prior to speaking to aid in the transcription of the dialogue following the session.

Interview Questions

1) Describe your philosophy of education with respect to either procedural fluency and/or conceptual understanding in mathematics?

2) What do you think is the best approach for incorporating instruction or activities that promote procedural fluency and/or conceptual understanding in your lessons?

3) As students learn mathematics, what is the natural flow of understanding and why do you think so?
    a. Conceptual understanding leads to procedural fluency
    b. Procedural fluency leads to conceptual understanding
    c. Either leads to either
    d. Other

4) What has been your experience with secondary (grades 7 to 12) mathematics curricula considering the concept of mathematical proficiency? Refer to the visual defining the strands of mathematical proficiency.

5) What are some of the experiences or examples that you have encountered in either training or practice that have lead to your approach to teaching as they pertain to procedural fluency and conceptual understanding?

APPENDIX F: VISUAL OF THE STRANDS OF MATHEMATICAL PROFICIENCY

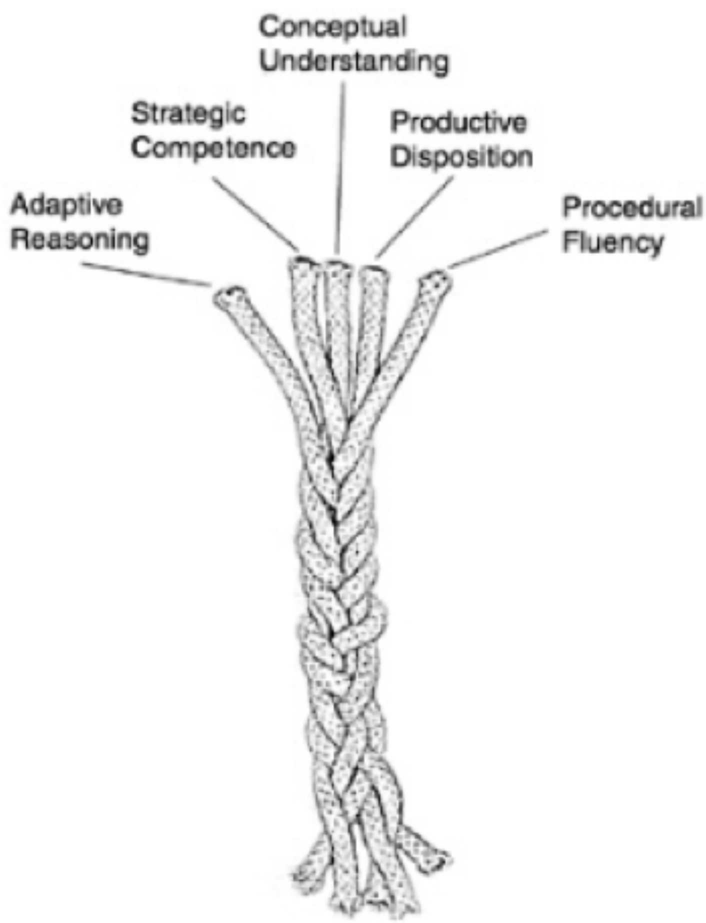
Conceptual Understanding – comprehension of mathematical concepts, operations, and relations

Procedural Fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

Strategic Competence – ability to formulate, represent, and solve mathematical problems

Adaptive Reasoning – capacity for logical thought, reflection, explanation, and justification

Productive Disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy



Kilpatrick, J., Swafford, J., Findell, B. & NRC (2001). *Adding it up: Helping children learn mathematics*. Washington DC: National Academy Press.

APPENDIX G: PERMISSION TO USE AMTE SURVEY

Brian

I am pleased that our survey might be of some use to you. Please feel free to use it.

Regarding validity and reliability issues, content validity was established as described by this statement:

"In preparation for a presentation at the Ninth Annual AMTE Conference in Dallas in January, 2005, the researchers in this study identified some issues related to the balance of conceptual and procedural learning and developed an initial survey to be used at the conference. Subsequent conference discussions and initial survey responses allowed for the refinement of questions associated with those issues and lead to the production of the current survey instrument."

Regarding reliability, you might consider gathering some data, perhaps preliminarily to your full-blown study, and then analyzing it.

Damon

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Mathematics Education
Department of Teacher Education
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801-422-6114

APPENDIX H: INVITATION TO PARTICIPATE IN SURVEY

E-mail Invitation

Dear Teacher,

My name is Brian Reilly. I am a doctoral student in Drexel University's Doctor of Education in Educational Leadership and Management program.

In partial fulfillment of my degree, I am studying secondary teachers' beliefs regarding mathematical proficiency in their instructional practices. I want to better understand how teachers design lessons, activities and assessments around students' conceptual understanding and/or procedural fluency of mathematical concepts focused on Algebra skills. If you are interested in being a part of this study, I would need approximately 20 minutes of your time to complete an online survey. The survey can be completed at any time and in any location that is most convenient for you.

Your participation in the study is completely voluntary. You may decline to answer any question or withdraw from the study at any time without consequence. Also, your confidentiality and privacy is extremely important to me. I will not report any identifiers or information that would identify you as an individual. The data will be reported in the aggregate and my report will be shared with my dissertation committee. Thereafter, I may publish or present my report publicly. If you have questions about this study, you may contact my supervising professor Dr. Fredricka Reisman at Freddie@drexel.edu or 215-895-6771. This research has been reviewed and approved by an Institutional Review Board (IRB) that ensures steps are taken to protect the rights and welfare of human subjects taking part in the research. You may contact them at HRPP@drexel.edu or 215-255-7857.

If you would like to participate in my study, please click on the title of the survey, Beliefs and Values for Teaching Mathematics Survey, which give you access to the online survey. Your submission of the online survey will serve as your implied consent to participate. If you know of other mathematics teachers teaching grades 7-12 that would be interested and willing to participate in this research study, please forward this invitation to them. You are under no obligation to share this information and whether or not you share this information will not reflect upon your own participation.

If you have any difficulty accessing the survey, please email me at bmr67@drexel.edu or contact me at 215-962-3511.

Thank you. Your participation is very much appreciated!

Brian

Brian Reilly
Drexel University
School of Education – Doctoral Candidate

APPENDIX I: INVITATION TO PARTICIPATE IN FOCUS GROUP

E-mail Invitation

Dear Teacher,

My name is Brian Reilly, and I am a doctoral student in Drexel University's Doctor of Education in Educational Leadership and Management program. While completing the survey: Beliefs and Values for Teaching Mathematics Survey conducted in the first part of my research study, you indicated your willingness to also participate in the second part of the study on secondary teachers' beliefs regarding mathematical proficiency in their instructional practices. In part two of the study, a focus group interview, I will be trying to better understand how teachers have developed their belief system, and how that belief system influences teachers' design of lessons, activities and assessments around students' conceptual understanding and/or procedural fluency of mathematical concepts focused on Algebra skills.

If you decide to participate, we will discuss your philosophy of education with a particular focus on mathematics and your perspectives on the relationship between curriculum and mathematical proficiency. The meeting will take place at a time and location that is mutually agreed upon by all participants, and should last about 60 minutes. The session will be audio recorded and video recorded so that I can accurately report and reflect on what is discussed. The recordings will only be reviewed by members of the research team who will transcribe and analyze them. There will be no personal identifiers used in the final reporting of the focus group interview in order to maintain strict confidentiality. The data collected will be kept in a secure location at Drexel University. The results of the study will be presented in my final doctoral dissertation.

The nature of a focus group interview does not guarantee complete privacy, as it is possible that other members of the group could share what they heard during the focus group session. By participating in the meeting you are indicating your willingness to be open and honest in your contributions, and you will also be respectful of the views and privacy of the other participants. Your participation in the study is completely voluntary. You may decline to answer any question or withdraw from the study at any time without consequence.

If you have questions about this study, you may contact my supervising professor Dr. Fredricka Reisman at Freddie@drexel.edu or 215-895-6771. This research has been reviewed and approved by an Institutional Review Board (IRB) that ensures steps are taken to protect the rights and welfare of human subjects taking part in the research. You may contact them at HRPP@drexel.edu or 215-255-7857.

If you would like to participate in this second phase of my study, please contact me at bmr67@drexel.edu or 215-962-3511 to discuss the specifics of the focus group meeting.

Thank you. Your participation is very much appreciated!
Brian
Brian Reilly
Drexel University
School of Education – Doctoral Candidate